Ziegler, Günter M.

Lectures on polytopes. (English) Zbl 0823.52002

Graduate Texts in Mathematics. 152. Berlin: Springer-Verlag. ix, 370 p. (1995).

This book introduces into the world of convex polytopes in d- dimensional affine spaces. It is developed from a course for advanced studies, and the reader can notice the author's successful didactical effort and enthusiasm for this subject.

Famous in the theory of polytopes is the classical treatment of *L. Schläfli* ['Theorie der vielfachen Kontinuität' (1850-1852), Gesammelte Math. Abh. Vol. 1, Birkhäuser Basel, pp. 167-387 (1949; Zbl 0035.21902)]. The modern theory was established by *B. Grünbaum* ['Convex polytopes', Interscience, London (1967; Zbl 0163.16603)]. Using these concepts and results here, now the author gives an excellent introduction to some basic methods and furthermore modern tools of polytope theory. Except for the basics in chapter 0 and 3 the lectures of the book are essentially independent from each other and require only a basic background knowledge of real affine geometry and of vector spaces. Especially, we find a concentration on combinatorial aspects of the theory. In every chapter the author shows important and interesting figures to illustrate his text, and he also tries to visualize the geometry of higher dimensional polytopes. At the end of each of the ten chapters there are interesting notations, problems, exercises, and historical comments. A good book to study geometry of polytopes and to give pleasure in this field! This expressed aim of the book has been doubtlessly achieved. It will be useful for both students and lecturers, for it can serve as an easy to read but nevertheless profound textbook on polytopes.

The titles of the ten chapters (and some subchapters) are: 0. Introduction and examples. 1. Polytopes, polyhedra and cones (Fourier- Motzkin elimination, Farkas lemma, Carathéodory's theorem). 2. Faces of polytopes (Face lattice, polarity, simplicial and simple polytopes). 3. Graphs of polytopes. 4. Steinitz' theorem for 3-polytopes. 5. Schlegel diagrams for 4-polytopes. 6. Duality, Gale diagrams and applications (Vector configurations, oriented matroids, polytopes with few vertices, rigidity, universality theorem). 7. Fans, arrangements, zonotopes, and tilings (Minkowski sums, nonrealizable oriented matroids, zonotopal tilings). 8. Shellability and the upper bound theorem (Euler-Poincaré formula, Dehn-Sommerville equations, upper bound theorem from McMullen, some extremal set theory, Kruskal-Katona theorem, Macaulay's theorem). 9. Fiber polytopes, and beyond.

At the end of the book we find references to nearly 500 titles on this subject.

Reviewer: J.Böhm (Jena)

MSC:

52-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to convex and discrete geometry
52A20 Convex sets in n dimensions (including convex hypersurfaces)
52Bxx Polytopes and polyhedra
05B35 Combinatorial aspects of matroids and geometric lattices

Keywords:

convex sets in n dimensions; affine space; polytopes; textbook

Software:

FourierMotzkin; LatticePolytopes; OldPolyhedra

Full Text: DOI