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Schneps, Leila

Dessins d'enfants on the Riemann sphere. (English) Zbl 0823.14017

Schneps, Leila (ed.), The Grothendieck theory of dessins d'enfants. Cambridge: Cambridge University Press. Lond. Math. Soc. Lect. Note Ser. 200, 47-77 (1994).

The paper under review is divided into four parts. Part I is devoted to an almost self-contained explanation of the bijection between the set of (abstract) clean dessins and the set of isomorphic classes of clean Belyi pairs (a clean Belyi pair (X, β) consists of an algebraic curve X defined over an algebraic closure $\overline{\mathbb{Q}}$ of $\mathbb{Q}$ and a holomorphic map $\beta : X \to \mathbb{P}^1(\mathbb{C})$ whose critical values lie in $\{0, 1, \infty\}$ such that all ramification orders over 1 are equal to 2). In part II, the action of the Galois group $G = \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on dessins of genus 1, and so on the profinite completion $\widehat{\pi}_1$ of the fundamental group of $\mathbb{P}^1(\mathbb{C}) - \{0, 1, \infty\}$ is studied. The main result of the section, attributed by the author to *H. W. Lenstra jun.* says that G acts also faithfully on trees, and so on genus 0 dessins. For the proof the author proves a couple of lemmas on the "uniqueness" of decomposition of univariate polynomials $F = G \circ H$ if $\deg(H)$ is fixed which were obtained before by *J. Gutierrez* and *C. Ruiz de Velasco* [in: Algebra and geometry, Proc. 2nd Span. Belg. Week, IISBWAG, Santiago de Compostela, Alxebra 54, 79-90 (1990; Zbl 0704.12003)].

In part III, the bijection of part I is made explicit for genus 0 dessins. Essentially, for a dessin D the method yields the set $O(D)$ of all dessins in the orbit of D under the action of G, the number field K_D, associated to each dessin D' in $O(D)$, a set of G- conjugate Belyi functions corresponding to the dessins in $O(D)$ and the action of G on $O(D)$. The procedure involves the computation of all solutions of a system of polynomial equations whose set of solutions is finite. To this purpose, Gröbner bases are used. – Finally, in part IV, numerical examples of this procedure are displaced.

The paper is very clearly written and provides a nonexpert people in the field, as the reviewer, an easy access to be subject.

For the entire collection see [Zbl 0798.00001].

Reviewer: Jose Manuel Gamboa (Madrid)

MSC:

14H57	Dessins d'enfants theory
11G32	Arithmetic aspects of dessins d'enfants, Belyĭ theory
30F99	Riemann surfaces
05E20	Group actions on designs, etc. (MSC2000)
14N10	Enumerative problems (combinatorial problems) in algebraic geometry

Cited in **1** Review
Cited in **35** Documents

Keywords:

clean dessins; clean Belyi pairs; action of the Galois group; Gröbner basis

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