

Liggett, Thomas M.; Petersen, Peter**The law of large numbers and $\sqrt{2}$.** (English) Zbl 0823.60023

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Let b_n be a positive sequence which satisfies

$$\lim_{n \rightarrow \infty} \sqrt[n]{b_n} \lambda > 0, \quad (*)$$

where $b_n^{(k)} = b_n^{(k-1)} + b_n^{(k-1)}$ and $b_n^{(0)} = b_n$. It is not too hard to observe that if $b_n^{(0)} = (1, 1, 2, 2, 4, 4, \dots)$, then $b_1^{(k)}/b_0^{(k)}$ approximates $\sqrt{2}$. The authors are interested in seeing how generally this procedure works. They show that under the above condition (*) it is true $\lim_{k \rightarrow \infty} b_1^{(k)}/b_0^{(k)} = \lambda$. The method of proof is based on an elementary version of the law of large numbers and a strengthened form of large deviations.

Reviewer: [N.G.Gamkrelidze \(Moskva\)](#)**MSC:**

60F15 Strong limit theorems

60F10 Large deviations

Keywords:[law of large numbers](#); [large deviations](#)**Full Text:** [DOI](#)