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Global solutions of two-dimensional Navier-Stokes and Euler equations. (English)

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Arch. Ration. Mech. Anal. 128, No. 4, 329-358 (1994).

The Cauchy problem for the Navier-Stokes system in $\mathbb{R}^2$ is considered in the vorticity formulation. A new approach to the existence and uniqueness results based upon comparison principles for linear parabolic equations is proposed. The existence of global solutions to Navier-Stokes (NS) and Euler (E) equations with the initial vorticity in $L^1(\mathbb{R}^2)$ for (NS) and in $(L^1 \cap L^r)(\mathbb{R}^2)$, $r > 2$, is proved. The solution to (NS) is shown to be unique, smooth and continuously dependent on initial data, and the velocity solution to (E) is Hölder continuous in the space and time coordinates. Another result is the existence of a subsequence of solutions to (NS) converging to a solution of (E) as the viscosity vanishes.

Reviewer: T.O.Shaposhnikova (Linköping)

MSC:

35Q30 Navier-Stokes equations
76D05 Navier-Stokes equations for incompressible viscous fluids
76B47 Vortex flows for incompressible inviscid fluids

Keywords:

Navier-Stokes equations; Euler equations; global solutions; Cauchy problem; existence and uniqueness results

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