Bar-Lev, Shaul K.; **Enis, Peter**; **Letac, Gérard**

Sampling models which admit a given general exponential family as a conjugate family of priors. (English) Zbl 0827.62002

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Summary: Let $\mathcal{K} = \{K_\lambda : \lambda \in \Lambda\}$ be a family of sampling distributions for the data x on a sample space $\mathcal{X}$ which is indexed by a parameter $\lambda \in \Lambda$, and let $\mathcal{F}$ be a family of priors on Λ. Then $\mathcal{F}$ is said to be conjugate for $\mathcal{K}$ if it is closed under sampling, that is, if the posterior distributions of λ given the data x belong to $\mathcal{F}$ for almost all x.

We set up a framework for the study of what we term the dual problem: for a given family of priors $\mathcal{F}$ (a subfamily of a general exponential family), find the class of sampling models $\mathcal{K}$ for which $\mathcal{F}$ is conjugate. In particular, we show that $\mathcal{K}$ must be a general exponential family dominated by some measure Q on $(\mathcal{X}, \mathcal{B})$, where $\mathcal{B}$ is the Borel field on $\mathcal{X}$. It is the class of such measures Q that we investigate. We study its geometric features and general structure and apply the results to some familiar examples.

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