Koch, Herbert; **Tataru, Daniel**

On the spectrum of hyperbolic semigroups. (English) Zbl 0823.35108

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The authors obtain a complete characterization of the Fredholm spectrum of the semigroup $S(t)$, generated by hyperbolic initial-boundary value problems, in terms of some quantities using certain scalar ordinary differential equations along the bicharacteristic flow for the operator P in the domain K. These are accompanied by sharp energy estimates on the norm of the semigroup above and below (whenever possible). Now we can state the main result.

Theorem. Assume that the coefficient of P are smooth, the functions g and h are smooth and either $g \equiv 0$ or Re $g < 0$, and for each $\gamma \in$ char P there exists a unique forward generalized bicharacteristic passing through γ. Assume that for the Riemannian manifold (Ω, α) does not exist an open set of closed generalized geodesics which have the same length T, for any T (in the opposite case similar results are obtained). Then outside the Fredholm spectrum, the spectrum of $S(t)$ can contain only isolated eigenvalues of finite multiplicity. The Fredholm spectrum is characterized as follows:

A. Assume that either the Dirichlet or the Neumann boundary condition is fulfilled and that $g(x) = 0$ in the latter case. Then

$$-\infty < \sigma_* \le \sigma^* \quad \Big(\sigma^* = \tfrac{1}{2} \lim_{t\to\infty} t^{-1} \sup_{\gamma_0} \omega(t-0, \gamma_0),\ \sigma_* = \tfrac{1}{2} \lim_{t\to\infty} t^{-1} \inf_{\gamma_0} \omega(t, \gamma_0)\Big) \quad \text{and}$$

$$F(S(t)) = \{z \in \mathbb{C};\ e^{\sigma_* t} \le |z| \le e^{\sigma^* t}\}, \quad \text{for any } t > 0.$$

B. Assume that the Neumann boundary condition is fulfilled with Re $g(x) < -1$ whenever Im $g(x) = 0$. Then $-\infty < \sigma_* \le \sigma^*$ and

$$F(S(t)) = \{0\} \cup \{z \in \mathbb{C};\ e^{\sigma_* t} \le |z| \le e^{\sigma^* t}, \quad \text{for any } t > 0.$$

C. Assume that the Neumann boundary condition is fulfilled and that $-1 \le$ Re $g(x) < 0$ and Im $g(x) = 0$ for some $x \in \partial K$. Then $-\infty = \sigma_* < \sigma^*$ and

$$F(S(t)) = \{z \in \mathbb{C};\ |z| \le e^{\sigma^* t}\}, \quad \text{for any } t > 0.$$

Thus, except for an at most countable set of values of t for a very restrictive class of Riemannian manifolds, the Fredholm spectrum of $S(t)$ is a full annulus (in case A), a full annulus and the origin (in case B) or a full disc (in case C).

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