Cignoli, Roberto

Free lattice-ordered abelian groups and varieties of MV-algebras. (English) Zbl 0827.06012

Proceedings of the IX Latin American symposium on mathematical logic, Bahía Blanca, Argentina, August 3-8, 1992. Part 1. Bahía Blanca: Universidad Nacional del Sur, Notas Logica Mat. 38, 113-118 (1993).

C. C. Chang introduced MV-algebras in 1958 as the Lindenbaum algebras of the infinite-valued sentential calculus of Lukasiewicz. Following the reviewer's paper "Interpretation of AF C^*-algebras in Lukasiewicz sentential calculus" [J. Funct. Anal. 65, 15-63 (1986; Zbl 0597.46059)], a structure $A = (A, 0, 1, ^*, \oplus)$ is called an MV-algebra iff $(A, 0, \oplus)$ is an abelian monoid, and $x \oplus 1 = 1$, $0^* = 1$, $1^* = 0$, $(x^* \oplus y)^* \oplus y = (y^* \oplus x)^* \oplus x$; in the same paper it is shown that MV-algebras are categorically equivalent to abelian lattice-ordered groups with strong unit. Chang proved that the variety of MV-algebras is generated by the rational unit interval $[0, 1]$ equipped with negation $1 - x$ and truncated addition: he used quantifier elimination for totally ordered divisible abelian groups. Using logic-syntactic machinery, Rose and Rosser had previously proved an equivalent version of this theorem. Very recently, *G. Panti* [J. Symb. Log. 60, No. 2, 563-578 (1995)] gave another proof using the De Concini Procesi theorem on elimination of points of indeterminacy in toric varieties. The author gives a simpler proof of the theorem using the above-mentioned categorical equivalence, together with Weinberg's well-known representation of free abelian lattice-groups as subdirect products of copies of the additive group $\mathbb{Z}$ of integers with natural ordering. (The first elementary proof of the theorem is forthcoming, jointly by Cignoli and the present reviewer, in a special issue of Studia Logica.) Using the completeness of the theory of certain classes of totally ordered abelian groups, Komori characterized all subvarieties of MV-algebras. The author proves the same result, in a much simpler way, again making use of Weinberg's representation theorem.

For the entire collection see [Zbl 0812.00017].

Reviewer: D.Mundici (Milano)

MSC:

06F20 Ordered abelian groups, Riesz groups, ordered linear spaces
06D30 De Morgan algebras, Łukasiewicz algebras (lattice-theoretic aspects)
03B50 Many-valued logic

Keywords:

Chang's completeness; Komori's classification; MV-algebra; ordered abelian groups; Weinberg's representation theorem