Arbogast, Todd; **Chen, Zhangxin**

On the implementation of mixed methods as nonconforming methods for second-order elliptic problems. (English) Zbl 0829.65127

Math. Comput. 64, No. 211, 943-972 (1995).

The authors show that mixed finite element methods (FEM) for a fairly general second-order elliptic problem with variable coefficients can be given in a nonmixed formulation. As lower-order terms are considered the results obtained can be applied to parabolic equations.

First of all, the authors begin with the development of a general theory on the equivalence of mixed and nonconforming methods. A nonconforming method which is defined for some finite element space is a Galerkin method with the addition of some special projection operators. Three conditions on this finite element space are developed that are sufficient to imply the equivalence of the new method to a given mixed method.

The problem how to construct finite element spaces that satisfy these three conditions is considered. For several sections, the authors restrict their attention to the lower-order Raviart-Thomas mixed method on rectangles. It is shown that the trivial postprocessing of the nonconforming solution obtained recovers the mixed solution and that the nonconforming solution is an approximation to the solution obtained by the mixed method by special postprocessing. This equivalence is exploited to derive optimal-order multigrid algorithms for the mixed and proposed nonconforming method.

The results mainly shown explicitly in the two space dimensions are extended to the three-dimensional case of the mixed methods defined over rectangular parallelepipeds, simplices and prisms.

The paper is interesting for numerical mathematicians dealing with FEM. There are no numerical experiments shown in the article but some applications can be expected.

Reviewer: K.Georgiev (Sofia)

MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N55 Multigrid methods; domain decomposition for boundary value problems involving PDEs
35J25 Boundary value problems for second-order elliptic equations

Keywords:

mixed finite element methods; second-order elliptic problem; variable coefficients; nonconforming methods; Galerkin method; multigrid algorithms

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