Martin, R.

Skew group rings and maximal orders. (English) Zbl 0830.16018

Glasg. Math. J. 37, No. 2, 249-263 (1995).

Let S be a prime Noetherian ring and let G be a finite group acting on S such that G is X-outer. Let $T = S * G$ be the skew group ring and let Ω_0 be the set of reflexive height-1 G-prime ideals of S. In the main theorem it is proved that if (a) S is a G-maximal order (i.e. an order which is not properly contained in any G-invariant order to which it is equivalent), and (b) p_0T is a prime ideal of T for all p_0 in Ω_0, then T is a prime maximal order. Conversely, if T is a (prime) maximal order and the order of G is a unit in S then (a) and (b) both hold. An example is given to show that the restriction on the order of G is necessary. In order to prove this theorem the author develops a theory of G-maximal orders analogous to that for maximal orders. Now let S be commutative and for each $1 \neq g \in G$ define $I(G)$ to be the ideal of S generated by the set $s - s^g$ $(s \in S)$. Then it is proved that T is a prime maximal order if and only if S is integrally closed and there does not exist $1 \neq g \in G$ and a height-1 prime ideal p in S such that $I(g) \subseteq p$. *E. Nauwelaerts* and *F. Van Oystaeyen* [J. Algebra 101, 61-68 (1986; Zbl 0588.16002)] have given sufficient conditions for a ring R strongly graded by a finite group G, with the order of G a unit in R, to be a tame order.

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MSC:

16S35	Twisted and skew group rings, crossed products	Cited in **8** Documents
16H05	Separable algebras (e.g., quaternion algebras, Azumaya algebras, etc.)	
16P40	Noetherian rings and modules (associative rings and algebras)	
16D25	Ideals in associative algebras	

Keywords:

prime Noetherian rings; finite groups; X-outer automorphisms; skew group rings; reflexive height-1 G-prime ideals; G-maximal orders; prime maximal orders; height-1 prime ideals; strongly graded rings; tame orders

Full Text: DOI

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