Hill, Theodore P.

The significant-digit phenomenon. (English) Zbl 0833.60003

Am. Math. Mon. 102, No. 4, 322-327 (1995).

The paper is a survey article on scale-invariance and base-invariance. In order to explain the significant-digit phenomenon (Benford's law) the author defines a natural domain $\mathcal{A}$ on the positive reals which is generated by infinite basis sets $S_{a,b} = \bigcup_{i=-\infty}^{\infty}[a10^i, b10^i)$, $1 \le a < b \le 10$. On $\mathcal{A}$ significant-digit laws to base 10 can be studied. A probability measure P is called scale-invariant if (i) $P(S) = P(\alpha S)$ for all real $\alpha > 0$ and all $S \in \mathcal{A}$. A probability measure P is called base-invariant if (ii) $P(S_{1,b}) = P(S_{1,b}^{1/k})$ for all positive integers k and all basis sets $S_{1,b}$, where $S^{1/k} = \{x^{1/k} : x \in S\}$ [cf. the author, Proc. Am. Math. Soc. 123, No. 3, 887-895 (1995; Zbl 0813.60002)]. Both scale-invariance and base-invariance imply Benford's law, essentially. The special role of the constant 1 is discussed.

Reviewer's remarks: 1. Under base-invariance of a probability measure p the defining relation (ii) holds for all $S \in \mathcal{A}$, not only for basis sets. 2. For the definition of scale-invariance of a probability measure P it suffices that the defining relation (i) is satisfied only for basis sets $S_{1,b}$. Then it holds for all $S \in \mathcal{A}$. After this, the definitions and properties of scale-invariance and base- invariance would look more alike. 3. The construction of $\mathcal{A}$ has the advantage of avoiding random variables in the definition of scale- invariance and base-invariance but the disadvantage of not explaining the simultaneous validity of Benford's law to several bases.

Reviewer: P.Schatte (Freiberg)

MSC:

60A10	Probabilistic measure theory
60E99	Distribution theory
11K16	Normal numbers, radix expansions, Pisot numbers, Salem numbers, good lattice points, etc.

Keywords:

significant-digit problem; Benford's law; scale-invariance; base- invariance

Full Text: DOI