Lancaster, Peter; Rodman, Leiba
Algebraic Riccati equations. (English) Zbl 0836.15005
Oxford: Clarendon Press. xvii, 480 p. (1995).

The growth of interest in algebraic Riccati equations in the last 35 years has been explosive. Primarily, this has been driven by the important role played by these equations in optimal filter design and control theory, and has resulted in a prodigious number of research publications which have steadily increased our understanding of the equations and their solution set. Although the algebraic Riccati equations are truly nonlinear, they are amenable to solutions by methods which rely heavily on linear algebra and the theory of matrices. Also, it transpires that a very important and pervasive role is played by rational matrix functions, their realizations and their factorization.

The book under review is divided into four parts. The first contains necessary material from linear algebra and the analysis of rational matrix functions. Canonical forms of matrices which are either self- adjoint or unitary in an indefinite scalar product play an important role and are the topic of Chapters 2 and 3. This particular collection of results has some novel; features and may be more widely useful. Not surprisingly, many properties of solutions of linear matrix equations are required and these are collected in Chapter 5. A necessarily lengthy discussion of rational matrix functions and their realizations appears in Chapter 6. This part also includes results on regular linear pencils of matrices. It is now well recognized that they play an important part in the analysis of algebraic Riccati equations and, in particular, in the study of numerical methods.

Part II and III are the heart of the monograph and concern the CARE and DARE, respectively. There are considerable parallels between the two cases. Indeed, a Cayley-technique allows some results to be transferred from one equation to the other. However, there are also substantial areas where independent treatments are necessary. In both cases solutions can be related to invariant subspaces of an operator formed from the coefficients of the equation, or to deflating subspaces of linear pencils, and these are primary tools used in Parts II and III. However, characterizations using rational matrix functions arise naturally when the transfer functions of the underlying time-invariant linear systems are introduced, and they form another theme running through these two parts.

Part IV contains accounts of several problem areas in which Riccati equations arise and the general theory developed in the Parts II and III is applied in each case. These chapters can be read independently and, with the exception of formal definitions, can be read independently of the rest of the book. They mostly form a part of a more general theory and the exposition may give the reader a foothold in that more general theory, as well as highlighting the special role played therein by an algebraic Riccati equation. Notes are appended to most chapters which give literature references, often in a historical context, for results developed in the chapters as well as closely related results.

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MSC:

15A24	Matrix equations and identities
15-02	Research exposition (monographs, survey articles) pertaining to linear algebra
15A22	Matrix pencils
93C05	Linear systems in control theory
93C99	Model systems in control theory
15A23	Factorization of matrices

Keywords:

extremal solution; canonical forms; algebraic Riccati equations; optimal filter design; control theory; rational matrix functions; factorization; indefinite scalar product; linear matrix equations; regular linear pencils of matrices; monograph; time-invariant linear systems