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Diophantine approximations and the sets of divergence of some Fourier series. (English. Russian original) Zbl 0863.42007

Russ. Acad. Sci., Dokl., Math. 49, No. 3, 471-473 (1994); translation from Dokl. Akad. Nauk, Ross. Akad. Nauk 336, No. 2, 151-153 (1994).

Let $f(x)$ be a continuous 2π-periodic function whose Fourier coefficients satisfy the condition

$$\sum_{n=1}^{\infty}(|a_n|+|b_n|)<\infty. \tag{1}$$

Let M denote the set of $\theta\in\mathbb{R}$ for which the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}f(n\theta)$ diverges and $h_\alpha(M)$ is the Hausdorff α-measure of M.

Theorem 1. Let the Fourier coefficients of the function $f(x)$ be monotone decreasing and satisfy (1). Then $h_\alpha(M)=0$ for any $\alpha>0$.

Theorem 2. There exist functions $f(x)$ with (1) such that $h_\alpha(M)>0$ for any $0<\alpha<\alpha_0(f)$.

The condition (1) is less stringent than the condition by Rao.

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42A20 Convergence and absolute convergence of Fourier and trigonometric series
11J71 Distribution modulo one

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