Demailly, Jean-Pierre; **Peternell, Thomas**; **Schneider, Michael**

Holomorphic line bundles with partially vanishing cohomology. (English) Zbl 0859.14005

Teicher, Mina (ed.), Proceedings of the Hirzebruch 65 conference on algebraic geometry, Bar-Ilan University, Ramat Gan, Israel, May 2-7, 1993. Ramat-Gan: Bar-Ilan University, Isr. Math. Conf. Proc. 9, 165-198 (1996).

Let X denote a complex manifold of dimension n. The authors study holomorphic line bundles L on X with partially vanishing cohomology (or having metrics with positive eigenvalues of curvature). They define $\sigma_+(L)$ to be the smallest integer q with the following property: There exists an ample divisor D on X and a constant $c > 0$ such that $H^j(X, mL - pD) = 0$ for all $j > q$ and $mp \geq 0$, $m \geq c(p+1)$. Note that $\sigma_+(L) = 0$ if and only if L is ample while $\sigma_+(L) = n$ if and only if $c_1(L^*)$ is in the closure of the cone of effective divisors. An ample q-flag is defined as a sequence $Y_q \subset Y_{q+1} \subset \cdots \subset Y_n = X$ of subvarieties Y_k of X such that $\dim Y_k = k$ and Y_k is the image of an ample Cartier divisor in the normalization of Y_{k+1}. Then a line bundle L is called q-flag positive if for some ample q-flag, $L \mid Y_q$ is positive.

Vanishing theorem: If $L \in \mathrm{Pic}X$ is q-flag positive then $\sigma_+(L) \leq n - q$.

The converse of this theorem is not true in general. A counter example and a positive result (of converse) for $\mathbb{P}_{n-1}$ bundles over a curve are given. The structure of projective 3-folds with $\sigma_+(-K_X) = 1$, $K_X =$ canonical bundle, is investigated. One has $\sigma_+(-K_X) = 0$ if and only if X is Fano and $\sigma_+(-K_X) \leq 2$ if and only if $\kappa(X) = -\infty$. The authors also study various cones in $NX(X) \otimes \mathbb{R}$, $NX(X)$ being Néron-Severi group, i.e. the group of divisors modulo numerical equivalence. All these cones coincide for surfaces.

For the entire collection see [Zbl 0828.00035].

Reviewer: U.N.Bhosle (Bombay)

MSC:

14F17 Vanishing theorems in algebraic geometry
32L20 Vanishing theorems
14F05 Sheaves, derived categories of sheaves, etc. (MSC2010)
14C22 Picard groups

Keywords:

flag; holomorphic line bundles; vanishing cohomology