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Asymptotic behaviour via the Harnack inequality. (English) Zbl 0840.35011

Ambrosetti, A. (ed.) et al., Nonlinear analysis. A tribute in honour of Giovanni Prodi. Pisa: Scuola Normale Superiore, Quaderni. Università di Pisa. 135-144 (1991).

Let $Lu = a_{ij}u_{ij} + b_iu_i + cu$ be uniformly elliptic with L^∞ coefficients. The authors investigate solutions of $Lu = 0$ on the semi-infinite cylinder $[0, \infty) \times \omega$, $\omega \subset \mathbb{R}^{n-1}$, with $\partial u / \partial \nu = 0$ on $[0, \infty) \times \partial \omega$. They show that if u, v are positive solutions with $u, v \rightarrow 0$ as $x_1 \rightarrow \infty$, and if $c(x) \leq 0$ then, for some constant $A > 0$, $v(x_1, y) / u(x_1, y) \rightarrow A$ as $x_1 \rightarrow \infty$, uniformly in ω . The same estimate is proved when v is as before and u satisfies the semilinear equation $Lu = f(x, u)$, provided $|f(x, u)| \leq Cu^{1+\delta}$ for some $\delta > 0$, $0 < u$ small, and $c(x) \leq -m < 0$. As a corollary, a similar asymptotic estimate is proved for solutions in \mathbb{R}^n when $|x|b_i(x)$ and $|x|^2c(x)$ are bounded for $|x| \geq 1$.

For the entire collection see Zbl 0830.00011.

Reviewer: [G.Porru \(Cagliari\)](#)

MSC:

- [35B40](#) Asymptotic behavior of solutions to PDEs
- [35J25](#) Boundary value problems for second-order elliptic equations
- [35J65](#) Nonlinear boundary value problems for linear elliptic equations

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Keywords:

[Harnack inequality](#); [semi-infinite cylinder](#); [positive solutions](#); [semilinear equation](#)