Nusse, Helena E.; **Yorke, James A.**
Wada basin boundaries and basin cells. (English) Zbl 0886.58072
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Summary: In dynamical systems examples are common in which two or more attractors coexist, and in such cases the basin boundary is nonempty. We consider a two-dimensional diffeomorphism F (that is, F is an invertible map and both F and its inverse are differentiable with continuous derivatives), which has at least three basins. Fractal basin boundaries contain infinitely many periodic points. Generally, only finitely many of these periodic points are "outermost" on the basin boundary, that is, "accessible" from a basin. For many systems, all accessible points lie on stable manifolds of periodic points. A point x on the basin boundary is a Wada point if every open neighborhood of x has a nonempty intersection with at least three different basins. We call the boundary of a basin a Wada basin boundary if all its points are Wada points. Our main goal is to have definitions and hypotheses for Wada basin boundaries that can be verified by computer. The basic notion "basin cell" will play a fundamental role in our results for numerical verifications. Assuming each accessible point on the boundary of a basin B is on the stable manifold of some periodic orbit, we show that ∂B is a Wada basin boundary if the unstable manifold of each of its accessible periodic orbits intersects at least three basins. In addition, we find conditions for basins $B_1, B_2, \ldots, B_N$ ($N \geq 3$) to satisfy $\partial B_1 = \partial B_2 = \cdots = \partial B_N$. Our results provide numerically verifiable conditions guaranteeing that the boundary of a basin is a Wada basin boundary. Our examples make use of an existing numerical procedure for finding the accessible periodic points on the basin boundary and another procedure for plotting stable and unstable manifolds to verify the existence of Wada basin boundaries.

MSC:

37D45	Strange attractors, chaotic dynamics of systems with hyperbolic behavior
37B99	Topological dynamics

Cited in **20** Documents

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