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Pointwise multiplication of Besov and Triebel-Lizorkin spaces. (English) Zbl 0839.46026

Math. Nachr. 175, 85-133 (1995).

Summary: It is shown that para-multiplication applies to a certain product $\pi(u,v)$ defined for appropriate u and v in $\mathcal{S}'(\mathbb{R}^n)$. Boundedness of $\pi(\cdot,\cdot)$ is investigated for the anisotropic Besov and Triebel-Lizorkin spaces – i.e., for $B^{M,s}_{p,q}$ and $F^{M,s}_{p,q}$ with $s\in\mathbb{R}$ and p and q in $]0,\infty]$ (though $p<\infty$ in the F-case) – with a treatment of the generic as well as of various borderline cases.

For $\max(s_0,s_1)>0$ the spaces $B^{M,s_0}_{p_0,q_0}\oplus B^{M,s_1}_{p_1,q_1}$ and $F^{M,s_0}_{p_0,q_0}\oplus F^{M,s_1}_{p_1,q_1}$ to which $\pi(\cdot,\cdot)$ applies are determined. For generic $F^{s_0}_{p_0,q_0}\oplus F^{s_1}_{p_1,q_1}$ the receiving $F^{s}_{p,q}$ spaces are characterized.

It is proved that $\pi(f,g)=f\cdot g$ holds for functions f and g when $f\cdot g\in L_{1,\mathrm{loc}}$ roughly speaking. In addition, $\pi(f,u)=fu$ when $f\in\mathcal{O}_M$ and $u\in\mathcal{S}'$.

Moreover, for an arbitrary open set $\Omega\subset\mathbb{R}^n$, a product $\pi_\Omega(\cdot,\cdot)$ is defined by lifting to $\mathbb{R}^n$. Boundedness of π on $\mathbb{R}^n$ is shown to carry over to π_Ω is general.

MSC:

46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems

46F05 Topological linear spaces of test functions, distributions and ultradistributions

Keywords:

boundedness; para-multiplication; anisotropic Besov and Triebel-Lizorkin spaces

Full Text: DOI

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