

**Allouche, Jean-Paul; Bousquet-Mélou, Mireille**

**On the conjectures of Rauzy and Shallit for infinite words.** (English) Zbl 0859.11019  
Commentat. Math. Univ. Carol. 36, No. 4, 705-711 (1995).

Shallit has conjectured the following result: Let  $(u_n)$  be an infinite sequence over a finite alphabet that is not ultimately periodic. Define  $S(n)$  to be the length of the longest suffix of  $u_0u_1 \dots u_n$  that is also a factor of  $u_0, u_1, \dots, u_{n-1}$ . Then

$$\liminf \frac{S(n)}{n} \leq C, \text{ where } C = \frac{3 - \sqrt{5}}{2}.$$

The authors show a connection between Shallit's conjecture and a conjecture due to Rauzy on the recurrence function  $R(n)$  of a non-ultimately periodic primitive sequence  $u$  ( $R(n)$  is the smallest integer such that every factor of  $u$  of length  $R(n)$  contains at least one occurrence of each factor of  $u$  of length  $n$ ). Rauzy's conjecture states that

$$\limsup_{n \rightarrow +\infty} \frac{R(n)}{n} \geq \frac{5 + \sqrt{5}}{2}.$$

In fact, Shallit's conjecture is proved to be equivalent to a Rauzy-like conjecture: if  $R'(n)$  denotes the length of the shortest prefix of the sequence  $u$  that contains at least one occurrence of each factor of  $u$  of length  $n$ , then we have:

$$\frac{1}{\liminf_{n \rightarrow +\infty} \frac{s(n)}{n}} = \limsup_{n \rightarrow +\infty} \frac{R'(n)}{n}.$$

Very recently, Julien Cassaigne has given a proof of Rauzy's conjecture and, by studying the Rauzy-like conjecture, has produced a counter-example to Shallit's conjecture, which seems to indicate that the constant  $C = \frac{3 - \sqrt{5}}{2}$  has to be replaced by  $\frac{29 - 2\sqrt{10}}{9}$ .

Reviewer: [V.Berthé \(Marseille\)](#)

**MSC:**

[11B85](#) Automata sequences  
[68R15](#) Combinatorics on words

Cited in **3** Documents

**Keywords:**

[infinite words](#); [Shallit's conjecture](#); [Rauzy's conjecture](#); [combinatorics on words](#); [length of the longest suffix](#); [recurrence function](#); [length of the shortest prefix](#)

**Full Text:** [EuDML](#)