Kirillov, A. A. jun.; **Etingof, P. I.**

A unified representation-theoretic approach to special functions. (English. Russian original)
Zbl 0868.33010

Funct. Anal. Appl. 28, No. 1, 73-76 (1994); translation from Funkts. Anal. Prilozh. 28, No. 1, 91-94 (1994).

The paper provides a new method of obtaining classes of special functions by means of the group-theoretic approach.

Let $\mathcal{H}$ be a Hopf algebra and $H \subset \mathcal{H}$ a subgroup of the group of invertible elements. Let U, V, W be irreducible $\mathcal{H}$-modules and $\varphi : V \to W \otimes U$ an intertwining operator for $\mathcal{H}$.

The functions $f_{vw\varphi}(h) = \langle w, \varphi hv\rangle \in U$ ($h \in H$, $v \in V$, $w \in W^*$) are called the U-valued matrix elements. In case of $V = W$ the function $\chi_\varphi(h) = \mathrm{Tr}|_V(\varphi h)$ is called the U-valued character.

The paper shows that in particular cases the above construction leads to (1) the usual matrix elements and character of group representations; (2) the Clebsch-Gordan coefficients; (3) the matrix elements of intertwining operators corresponding to representations of quantum affine Lie groups.

The authors give the generalized Peter-Weyl theorem for the vector-valued elements and characters of finite groups and compact Lie groups.

The most important applications correspond to $\mathcal{H}$ equal to the convolution algebra of distributions on a Lie group G and to $\mathcal{H}$ equal to the quantum affine algebra, $U_q(\widehat{\mathfrak{g}})$ for $\mathfrak{g}$ a simple Lie algebra. Three particular cases are worked out with details.

Reviewer: A.Wawrzynczyk (MR 95h:33010)

MSC:

33C80	Connections of hypergeometric functions with groups and algebras, and related topics
16W30	Hopf algebras (associative rings and algebras) (MSC2000)
17B37	Quantum groups (quantized enveloping algebras) and related deformations
81R30	Coherent states

Keywords:
Hopf algebra; Clebsch-Gordan coefficients

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