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Progrès récents de l'algèbre locale. (French) [Zbl 0086.25604](#)

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Progress in local algebra after the writing of the author's monograph "Algèbre locale" [Mém. Sci. Math. No. 123. Paris: Gauthier-Villars (1953; [Zbl 0053.01901](#))] are reviewed. The first topic is that of "modulation", which is discussed in connection with completion, zero divisors, Hensel's lemma [cf. the author, Proc. Int. Congr. Math. 1954 Amsterdam ([Zbl 0077.00303](#))] and Hilbert's characteristic polynomials. Next the lemma of Artin and Rees [*D. Rees*, Proc. Camb. Philos. Soc. 52, 155–157 (1956; [Zbl 0074.26201](#))] about an asymptotic property of modules is discussed in the context of the topology induced in a submodule, Krull's intersection and principal ideal theorems and the equivalence of different definitions of the dimension of a local ring. Then the introduction of methods of homological algebra is told, as the most substantial methodological recent progress in local algebra. Examples are *J.-P. Serre's* homological characterization of regular local rings [Proc. Int. Symp. Algebraic Number Theory 1955, 175–189 (1956; [Zbl 0073.26004](#))], its application to the regularity of quotient rings, and the definition of intersection multiplicities as certain Euler characteristics, also due to Serre. The Chevalley-Samuel-Nagata-Serre associativity formula for multiplicities is mentioned also in connection of homological algebra and *C. Lech's* new proof [Ark. Mat. 3, 301–314 (1957; [Zbl 0089.26002](#))]. Further, Nagata's contributions to many problems on local algebra, such as Zariski's theorem of irreducibility and analytic normality, Noetherian nature of integral closures, chains of prime ideals, multiplicities, Henselian, rings, etc. are alluded to together with those by Northcott-Rees to quadratic transformations, characteristic polynomials, etc. and the simple proofs of Cohen's theorem, on the structure of complete local rings, by *A. Geddes* [J. Lond. Math. Soc. 29, 334–341 (1954; [Zbl 0056.02904](#))] and *M. Narita* [Proc. Int. Symp. Algebraic Number Theory 1955, 251–253 (1956; [Zbl 0074.02803](#))]. The article closes with the proposition of several problems in local algebra.

For the entire collection see [[Zbl 0084.01202](#)].

Reviewer: T. Nakayama

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MSC:

[13Hxx](#) Local rings and semilocal rings

[13-02](#) Research exposition (monographs, survey articles) pertaining to commutative algebra

Keywords:

[progress in local algebra](#)