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**Another note on Levine's decomposition of continuity.** (English) Zbl 0845.54009  
Int. J. Math. Math. Sci. 19, No. 2, 317-320 (1996).

A subset  $A$  of a topological space  $X$  is  $\alpha$ -open if  $A \subset \text{Int}(\text{Cl}(\text{Int}(A)))$ . The collection of  $\alpha$ -open subsets of  $X$ , written  $X^\alpha$ , is a topology for  $X$  [*O. Njåstad*, Pac. J. Math. 15, 961-970 (1965; [Zbl 0137.41903](#))]. Let  $X$  and  $Y$  be two topological spaces. A function  $f : X \rightarrow Y$  is weakly continuous at  $x \in X$  [*N. Levine*, Am. Math. Mon. 68, 44-46 (1961; [Zbl 0100.18601](#))] if for any open set  $V \subset Y$  containing  $f(x)$  there exists an open set  $U \subset X$  containing  $x$  such that  $f(U) \subset \text{Cl}(V)$ . If this condition is satisfied at each  $x \in X$ , then  $f$  is said to be weakly continuous. A function  $f : X \rightarrow Y$  is weakly  $\alpha$ -continuous [*T. Noiri*, Int. J. Math. Math. Sci. 10, 483-490 (1987; [Zbl 0638.54012](#))] if  $f : X^\alpha \rightarrow Y$  is weakly continuous.

Two new function conditions (\*) and (\*\*) are introduced each strictly weaker than local relative continuity [the first and the second author, Real Anal. Exch. 20, 823-830 (1995; [Zbl 0838.54011](#))] and (\*\*) being strictly weaker than (\*).

**Definition 1.** A function  $f : X \rightarrow Y$  satisfies condition (\*) if for each  $x \in X$  and for each open set  $V$  containing  $f(x)$ , there exists an open set  $V_x \subset V$  with  $f(x) \in V_x$  such that  $x \notin \text{Cl}(f^{-1}(\text{Cl}(V_x) - f^{-1}(V)))$ .

**Definition 2.** A function  $f : X \rightarrow Y$  satisfies condition (\*\*) if for each open set  $V$  containing  $f(x)$ , there exists an open set  $V_x \subset V$  with  $f(x) \in V_x$  such that  $x \notin \text{Cl}(\text{Int}(f^{-1}(\text{Cl}(V_x))) - f^{-1}(V))$ .

It is shown that for any function  $f : X \rightarrow Y$ , the following are equivalent: (a)  $f$  is continuous; (b)  $f$  is weakly continuous and satisfies (\*\*); (c)  $f$  is weakly  $\alpha$ -continuous and satisfies (\*).

Decomposition (b) improves a result of *J. Chew* and *Tong Jingcheng* [Am. Math. Mon. 98, 931-934 (1991; [Zbl 0764.54007](#))] and Theorem 5 of [the first and the second author, loc. cit.]. Decomposition (c) improves a result of *Noiri* and Theorem 6 of [the first and the second author, loc. cit.].

Reviewer: [V.Popa \(Bacau\)](#)

**MSC:**

[54C10](#) Special maps on topological spaces (open, closed, perfect, etc.)  
[54C08](#) Weak and generalized continuity

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decomposition of continuity; weak continuity; local relative continuity

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