Rahola, Jussi

Diagonal forms of the translation operators in the fast multipole algorithm for scattering problems. (English) Zbl 0854.65122

BIT 36, No. 2, 333-358 (1996).

The integral equations of acoustic and electromagnetic scattering generate large dense systems of linear equations. These systems are solved by iterative methods, where the matrix-vector multiplication is computed using a special fast method. The author applies the fast multipole method using potential expansions of the dipole potential in terms of spherical Bessel functions, Legendre polynomials and spherical harmonics. The proposed method is based on the manipulation of truncated potential expansions.

Two kinds of errors are introduced in the considered method. Error analysis is given, considering both the truncation error of potential expansions and the errors from the use of numerical integration.

Reviewer: L.Hącia (Poznań)

MSC:

65R20 Numerical methods for integral equations
45E10 Integral equations of the convolution type (Abel, Picard, Toeplitz and Wiener-Hopf type)

Keywords:

error analysis; integral equations; acoustic and electromagnetic scattering; iterative methods; fast multipole method; spherical Bessel functions; Legendre polynomials; spherical harmonics; truncated potential expansions

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