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The differential Galois theory of regular singular p-adic differential equations. (English)
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Math. Ann. 305, No. 1, 45-64 (1996).

Let k be a perfect field of characteristic $p > 0$, X a smooth curve over k and suppose that there is a lifting X_K of X to an algebraic curve over a field K of characteristic zero. Let M be a locally free sheaf with connection on X_K satisfying some convergence condition (namely it is "soluble" in generic disks). Then, by restriction to strict neighborhoods of a formally smooth lifting of X, M defines an "overconvergent isocrystal" $M^\dagger$ on X. If X_K has a K-rational point x, the category of overconvergent isocrystals and the category of locally free sheaves on X_K with connection are neutral Tannakian categories. In both situations, the fiber functor associated to x restricted to the smallest tensor subcategory containing M (resp. $M^\dagger$) enables to define the "monodromy group" $\mathrm{DGal}(M)$ (resp. $\mathrm{DGal}(M^\dagger)$). The main result of the paper is that if $M^\dagger$ is regular, with p-adic integers exponents two of which do not differ by a p-adic Liouville number, then $\mathrm{DGal}(M)$ and $\mathrm{DGAL}(M^\dagger)$ are isomorphic.

As applications, first the unicity, up to homothety, of the Frobenius structure for irreducible F-isocrystals is shown, secondly a comparison result between monodromy groups of some isocrystals arising from geometry (Gauss-Manin connections) and corresponding ℓ-adic monodromy groups is established. Proofs are based on the so-called "transfer theorem for regular singular p-adic differential equations". The author gives two conditions for a locally free sheaf to be overconvergent. Let us point out that, at least over a curve, the second one is implied by the first one.

Reviewer: G.Christol (Paris)

MSC:

14F30	p-adic cohomology, crystalline cohomology	Cited in **1** Document
12H25	p-adic differential equations	
32S40	Monodromy; relations with differential equations and D-modules (complex-analytic aspects)	
18E30	Derived categories, triangulated categories (MSC2010)	

Keywords:

transfer theorem for regular singular p-adic different equations; overconvergent isocrystals; Tannakian categories

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