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Stability and delays in a predator-prey system. (English) Zbl 0873.34062

J. Math. Anal. Appl. 198, No. 2, 355-370 (1996).

This paper considers the following system of delay differential equations

$$\frac{dN_1}{dt} = N_1(t)[a_1 - b_1N_1(t-\tau) - c_1N_2(t-\sigma)],$$
$$\frac{dN_2}{dt} = N_2(t)[-a_2 + c_2N_1(t-\sigma) - b_2N_2(t)],$$

which models a predator-prey Lotka-Volterra system, with N_1 as the density of prey and N_2 as the predator, a_1, b_1 and the c_1 being positive constants. The delay σ in the first equation is justified by the author by "the fact that predators cannot hunt prey when the predators are infants", while in the second equation it corresponds to the time it takes for the predator to transform the prey into predator biomass. As noted by the author, there is no reason that the time to hunting maturity be the same as the time to predator biomass production: this choice is just made for simplicity. τ corresponds to delay in the negative feedback of the prey. Under some restrictions on the size of the delays, it is shown that the unique positive equilibrium is locally asymptotically stable. Under further restrictions on the delays, global asymptotic stability is proved. The proof is done by constructing a Lyapunov function, for the linearized equation first, then for the whole equation. The paper follows a previous work by *W. Wang* and *Z. Ma* [J. Math. Anal. Appl. 158, No. 1, 256-268 (1991; Zbl 0731.34085)]. The paper by Wang and Ma allows for more general delays. On the other hand, it only deals with uniform persistence, and proves that under the assumption on the coefficients of the system ensuring global asymptotic stability of the positive equilibrium of the system without delays, uniform persistence holds independent on the delays. Finally, a wrong statement, appeared in a paper by *K. N. Murty* and *M. A. S. Srinivas* [J. Math. Anal. Appl. 158, No. 2, 333-341 (1991; Zbl 0725.92025)], is pointed out and corrected.

Reviewer: O.Arino (Pau)

MSC:

34K20 Stability theory of functional-differential equations
92D25 Population dynamics (general)

Keywords:

delay differential system; predator-prey Lotka-Volterra system; global asymptotic stability; Lyapunov function; uniform persistence; positive equilibrium

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