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Weighted Sobolev and Poincaré inequalities and quasiregular mappings of polynomial type.

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Let $f:\mathbb{R}^n\to\mathbb{R}^n$ be a quasiregular mapping. The authors prove that the weight $w(x)=J(x,f)^{1-p/n}$ is p-admissible if and only if f is of polynomial type. The map f is said to be of polynomial type if $|f(x)|\to\infty$. Recall also that a weight $w(x)$ is p-admissible if it satisfies a weighted Sobolev inequality, a weighted Poincaré inequality and a doubling condition and if the gradient is unique in the relevant weighted Sobolev space. (See the first author, *T. Kilpelainen* and *O. Martio*, Nonlinear potential theory of degenerate elliptic equations (1993; Zbl 0780.31001).) These four conditions are needed in order to use the Moser iteration method for degenerate elliptic equations. The admissibility of this particular class of weights settles a question of *B. Øksendal* [Comm. partial differential equations 15, 1447-1459 (1990; Zbl 0719.31002)].

The authors set forth by first characterizing quasiregular maps of polynomial type in terms of six equivalent conditions. We mention only three: (i) f is of polynomial type, (ii) $J(x,f)$ is a doubling weight and (iii) $J(x,f)$ is a strong A_∞-weight. (See *G. David* and *S. Semmes*, Analysis and partial differential equations, Lect. Notes Pure Appl. Math. 122, 101-111 (1990; Zbl 0752.46014).) As an application of this result, they show that the image of a ball under a quasiregular map $f:\mathbb{R}^n\to\mathbb{R}^n$ of polynomial type is a John domain. In their proofs of the Sobolev and Poincaré inequality for the weight $w(x)=J(x,f)^{1-p/n}$, the authors present a different more elementary approach. This proof centers on the strong A_∞-weight characterization of the Jacobian of the quasiregular map.

Reviewer: S.Staples (Fort Worth)

MSC:

30C65	Quasiconformal mappings in $\mathbb{R}^n$, other generalizations
35J70	Degenerate elliptic equations
46E35	Sobolev spaces and other spaces of “smooth” functions, embedding theorems, trace theorems

Keywords:

quasiregular mapping; weighted Sobolev inequality; weighted Poincaré inequality; degenerate elliptic equations; strong A_∞-weight

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