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Asymptotic behaviour of Stieltjes polynomials for ultraspherical weight functions. (English)

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J. Comput. Appl. Math. 65, No. 1-3, 135-144 (1995).

This paper is concerned with the Stieltjes polynomials $E_{n+1}^{(\lambda)}(x)$ defined by

$$\int_{-1}^{1} w_\lambda(x)P_n^{(\lambda)}(x)E_{n+1}^{(\lambda)}(x)x^k\,dx = 0, \quad k = 0,1,2,\dots,n,$$

where $w_\lambda(x) = (1-x^2)^{\lambda-1/2}$, $\lambda > -1/2$, is the ultraspherical weight function and $P_n^{(\lambda)}(x)$ are the ultraspherical polynomials. The author gives an asymptotic representation of $E_{n+1}^{(\lambda)}(\cos\vartheta)$, as $n\to\infty$, in the case $1<\lambda\le 2$. This representation, which holds uniformly for $\varepsilon\le\vartheta\le\pi-\varepsilon$, $\varepsilon>0$, completes the study of the ultraspherical weight functions for which Stieltjes polynomials are known to have only real distinct zeros inside $(-1,1)$ for all n. The asymptotic behaviour obtained is successfully applied in proving interesting positivity results for Kronrod extensions of Gauss and Lobatto quadrature rules.

Reviewer: L.Gatteschi (Torino)

MSC:

42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
33C55 Spherical harmonics
41A55 Approximate quadratures

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approximate quadrature; Stieltjes polynomials; ultraspherical weight function; ultraspherical polynomials; asymptotic representation; Kronrod extensions; Gauss and Lobatto quadrature rules

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