Durany, J.; **García, G.**; **Vázquez, C.**

A mixed Dirichlet-Neumann problem for a nonlinear Reynolds equation in elastohydrodynamic piezoviscous lubrication. (English) Zbl 0857.35044

Proc. Edinb. Math. Soc., II. Ser. 39, No. 1, 151-162 (1996).

Given a rectangular $\Omega = \{(x_1, x_2) : |x_i| < 1\}$, the first goal is to find a nonnegative weak solution $p \in W^{1,2}(\Omega)$ of the equation

$$\operatorname{div}(h^3[p]\exp(-\alpha p)\nabla p) = \frac{\partial}{\partial x_1} c(H_\varepsilon(p)h[p])$$

with the boundary conditions $p = 0$ at $\{|x_2| = 1\} \cup \{x_1 = 1\}$ and $\frac{\partial}{\partial x_1} p = c(H_\varepsilon(p) - \theta_0)$ at $\{x_1 = -1\}$. Here c and α are positive constants, $h[p] = h_0(x) + p * 1/|x|$, h_0 is a smooth positive function, $H_\varepsilon(p)$, $\varepsilon \to 0$, is an approximation of the Heaviside function. Then the estimates $|p|_{L^\infty} \leq c$ and $|p|_{W^{1,2}} \leq c$ are obtained uniformly in ε to prove the convergence $p \to \overline{p}$ to a weak solution $\overline{p}$ of the limit equation with $H_\varepsilon(p)$ substituted by $\theta(x, t) = w\text{-}\lim H_\varepsilon(p)$. It is claimed that the pair $\overline{p}$ and θ solves the lubricant pressure problem modelled by the Reynolds equation.

Reviewer: V.Shelukhin (Novosibirsk)

MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
76D08 Lubrication theory
35R35 Free boundary problems for PDEs

Keywords:

Schauder's fixed point theorem; nonnegative weak solution

Full Text: DOI

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