Huxley, M. N.

Area, lattice points, and exponential sums. (English) Zbl 0861.11002

London Mathematical Society Monographs. New Series. 13. Oxford: Clarendon Press. xii, 494 p. (1996).

This book gives a very detailed exposition of exponential sum estimates and its applications to lattice point theory. Let a closed, sufficiently smooth curve in the plane with area A be given. Expanding the curve by a factor M there will be nearly AM^2 lattice points within the curve. The central problem in lattice point theory is to estimate the error term "number of lattice points within the curve minus area AM^2" as well as possible. A similar problem is to ask for the number of integer points which lie on or very close to a curve. If one uses Fourier theory the curve $y = F(x)$ leads to an exponential sum like $\sum \exp 2\pi i F(n)$. The theory of estimating exponential sums goes back to the beginning of our century. However, in 1986 E. Bombieri and H. Iwaniec found a very important new method for estimating exponential sums. Several mathematicians, especially the author, generalized the method and found various improvements. The central theme of this book is to give a detailed description of this method and to prove the best possible estimations of the error terms on lattice point theory.

The book is divided into six parts. The first part is quite elementary. The author proves van der Corput's theorem on the number of lattice points inside a convex region, and presents H. P. F. Swinnerton-Dyer's method of counting integer points close to a curve. In the next three parts one can find an extensive and very precise description of the Bombieri-Iwaniec method. Part V contains the applications: The new estimations of exponential sums due to the author; the Dirichlet divisor problem; lattice points in closed planar domains; the order of magnitude of the Riemann zeta function; mean squares; the twelfth-power moment; prime numbers in a smooth sequence. Part VI is devoted to related work and further ideas.

The book is very well written. It is an excellent and important work for all mathematicians who deal with exponential sums and lattice point theory. It is accessible to graduate students beginning research.

Reviewer: E.Krätzel (Wien)

MSC:

11-02	Research exposition (monographs, survey articles) pertaining to number theory
11P21	Lattice points in specified regions
11C99	Polynomials and matrices
11L07	Estimates on exponential sums
11H99	Geometry of numbers
11N37	Asymptotic results on arithmetic functions
11M06	$\zeta(s)$ and $L(s,\chi)$

Keywords:
estimates of exponential sums; lattice points in alrge plane domains; circle problem; error terms; van der Corput's theorem; number of lattice points inside a convex region; integer points close to a curve; Bombieri-Iwaniec method; Dirichlet divisor problem; lattice points in closed planar domains; Riemann zeta function; mean squares; twelfth-power moment; prime numbers in a smooth sequence