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Conjugacy classes in loop groups and G-bundles on elliptic curves. (English) Zbl 0992.20034
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Authors' introduction: Let $\mathbb{C}[[z]]$ be the ring of formal power series and $\mathbb{C}((z))$ the field of formal Laurent power series, the field of fractions of $\mathbb{C}[[z]]$. Given a complex algebraic group G, we will write $G((z))$ for the group of $\mathbb{C}((z))$-rational points of G, thought of as a formal "loop group", and $a(z)$ for an element of $G((z))$. Let q be a fixed nonzero complex number. Define a "twisted" conjugation action of $G((z))$ on itself by the formula

$$g(z) : a(z) \mapsto {}^g a = g(q \cdot z) \cdot a(z) \cdot g(z)^{-1}.$$

We are concerned with the problem of classifying the orbits of the twisted conjugation action. If $q = 1$, twisted conjugation becomes the ordinary conjugation, and the problem reduces to the classification of conjugacy classes in $G((z))$.

In this paper we are interested in the case $|q| < 1$. Let $G[[z]] \subset G((z))$ be the subgroup of $\mathbb{C}[[z]]$-points of G. A twisted conjugacy class in $G((z))$ is called integral if it contains an element of $G[[z]]$. Introduce the elliptic curve $\mathcal{E} = \mathbb{C}^*/q^{\mathbb{Z}}$.

Our main result is the following. Theorem 1.2. Let G be a complex connected semisimple algebraic group. Then there is a natural bijection between the set of integral twisted conjugacy classes in $G((z))$ and the set of isomorphism classes of semistable holomorphic principal G-bundles on $\mathcal{E}$.

MSC:

20G20	Linear algebraic groups over the reals, the complexes, the quaternions
14F05	Sheaves, derived categories of sheaves, etc. (MSC2010)
14H52	Elliptic curves
20E45	Conjugacy classes for groups

Keywords:
complex algebraic groups; orbits; twisted conjugation actions; twisted conjugacy classes; elliptic curves; connected semisimple algebraic groups; semistable holomorphic principal bundles

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