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Completion of topological loops. (English) [Zbl 0861.22002](#)
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Summary: A Hausdorff topological group equipped with the right uniformity admits a group completion iff the inversion mapping preserves Cauchy filters [cf. *N. Bourbaki*, *Topologie général* (Paris 1971; [Zbl 0249.54001](#))], III. §3, No. 5, Théorème 1]. Up until today a general theorem on the completion of topological loops is not available, for partial results see *H. Wefelscheid* [*Math. Z.* 99, 279-298 (1967; [Zbl 0153.35901](#))]. This is among others due to the fact that topological loops will not necessarily have a compatible right uniformity. The main results of this paper are the following: All topological loops are locally uniform in the sense of *J. Williams* [*Trans. Am. Math. Soc.* 168, 435-469 (1972; [Zbl 0235.54026](#))], and, provided the notion of “Cauchy filter” is suitably chosen, they can be completed. An analogue of the completion theorem for groups cited above holds for topological loops. According to these aims the theory of completion of locally uniform spaces is developed in sections 1-5 of this paper.

MSC:

[22A30](#) Other topological algebraic systems and their representations

Keywords:

Hausdorff topological group; group completion; Cauchy filters; topological loops; locally uniform spaces

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References:

- [1] N. Bourbaki, *Topologie générale*. Hermann, Paris, 1971. · [Zbl 0249.54001](#)
- [2] P. Fletcher, W. F. Lindgren, *Quasiuniform spaces*. Dekker, New York, 1982.
- [3] Th. Grundhöfer, H. Salzmann, Locally compact double loops and ternary fields, Ch. XI in O. Chein et al. (eds.), *Quasigroups and loops: theory and applications*, Heldermann, Berlin, 1990. · [Zbl 0749.51016](#)
- [4] K. H. Hofmann, Topologische Loops. *Math. Zeitschr.* 70 (1959), 13–37. · [Zbl 0095.02701](#) · [doi:10.1007/BF01558574](#)
- [5] K. H. Hofmann, K. Strambach, Topological and analytic loops, Ch. IX in O. Chein et al. (eds.), *Quasigroups and loops: theory and applications*, Heldermann, Berlin, 1990. · [Zbl 0747.22004](#)
- [6] H. Pflugfelder, *Quasigroups and loops: Introduction*. Heldermann, Berlin, 1990. · [Zbl 0715.20043](#)
- [7] H. Salzmann, Topological planes. *Adv. Math.* 2 (1967), 1–60. · [Zbl 0153.21601](#) · [doi:10.1016/S0001-8708\(67\)80002-1](#)
- [8] T. P. Ufnarovskaja, Uniform structures of IP-loops (Russian). *Bul Akad. Štiinca RSS Moldoven.* 1 (1972), 28–31, *Math. Review* 45, \# 6971. · [Zbl 0238.22004](#)
- [9] H. Wefelscheid, *Vervollständigung topologisch-algebraischer Strukturen*. Dissertation, Universität Hamburg, 1966.
- [10] H. Wefelscheid, *Vervollständigung topologischer Fastkörper*. *Math. Zeitschr.* 99 (1967), 279–298. · [Zbl 0153.35901](#) · [doi:10.1007/BF01181727](#)
- [11] J. Williams, Locally uniform spaces. *Trans. Amer. Math. Soc.* 168 (1972), 435–469. · [Zbl 0221.54022](#) · [doi:10.1090/S0002-9947-1972-0296891-5](#)

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