Liggett, Thomas M.

Multiple transition points for the contact process on the binary tree. (English) Zbl 0871.60087

Ann. Probab. 24, No. 4, 1675-1710 (1996).

Let T^d denote the homogeneous (connected) tree in which each vertex has $d+1$ neighbours and let (A_t) be the finite contact process on T_d. By definition, we have the transitions $A \to A \setminus \{x\}$ for $x \in A$ at rate 1, and $A \to A \cup \{x\}$ for $x \notin A$ at rate $\lambda \cdot \#\{y \in A : |y-x| = 1\}$ ($|y-x|$ denoting the distance between $x, y \in T^d$). (A_t) is said to survive strongly if $P^{\{x\}}(x \in A_t$ for arbitrarily large $t) > 0$. On the other hand, (A_t) survives if $P^{\{x\}}(A_t \neq 0,\ t \geq 0) > 0$. One says that (A_t) dies out if it does not survive, and that it survives weakly if it survives, but does not survive strongly. Critical values $\lambda_1 \leq \lambda_2$ are defined by the requirement that (A_t) survives strongly for $\lambda_1 > \lambda_2$, survives weakly for $\lambda_1 < \lambda < \lambda_2$ and dies out for $\lambda < \lambda_1$. In the case $d \geq 3$, *R. Pemantle* [ibid. 20, No. 4, 2089-2116 (1992; Zbl 0762.60098)] obtained upper bounds on λ_1 and lower bounds on λ_2 implying that $\lambda_1 < \lambda_2$. In the present paper it is shown that (for homogeneous trees) in the case $d = 2$, $\lambda_1 \leq 0.605$, $\lambda_2 \geq 0.609$ which implies $\lambda_1 < \lambda_2$.

Reviewer: K.Schürger (Bonn)

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory

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paper as accurately as possible without claiming the completeness or perfect precision of the matching.