

**Chaudhuri, Probal**

**On a geometric notion of quantiles for multivariate data.** (English) Zbl 0869.62040  
*J. Am. Stat. Assoc.* 91, No. 434, 862-872 (1996).

Summary: An extension of the concept of quantiles in multidimensions that uses the geometry of multivariate data clouds has been considered. The approach is based on blending as well as generalization of the key ideas used in the construction of spatial median and regression quantiles, both of which have been extensively studied in the literature. These geometric quantiles are potentially useful in constructing trimmed multivariate means as well as many other  $L$  estimates of multivariate location, and they lead to a directional notion of central and extreme points in a multidimensional setup. Such quantiles can be defined as meaningful and natural objects even in infinite-dimensional Hilbert and Banach spaces, and they yield an effective generalization of quantile regression in multiresponse linear model problems.

Desirable equivariance properties are shown to hold for these multivariate quantiles, and issues related to their computation for data in finite-dimensional spaces are discussed.  $n^{1/2}$  consistency and asymptotic normality of sample geometric quantiles estimating the corresponding population quantiles are established after deriving a Bahadur-type linear expansion. The sampling variation of geometric quantiles is carefully investigated, and estimates for dispersion matrices, which may be used in developing confidence ellipsoids, are constructed. In course of this development of sampling distributions and related statistical properties, we observe several interesting facts, some of which are quite counterintuitive. In particular, many of the intriguing properties of spatial medians documented in the literature appear to be inherited by geometric quantiles.

Reviewer: [Reviewer \(Berlin\)](#)

**MSC:**

**62H05** Characterization and structure theory for multivariate probability distributions; copulas

Cited in **3** Reviews  
Cited in **96** Documents

**Keywords:**

*L* estimates of multivariate location; Bahadur representation; multiresponse quantile regression; geometry of multivariate data clouds; spatial median; regression quantiles; geometric quantiles; trimmed multivariate means; equivariance properties; consistency; asymptotic normality; confidence ellipsoids

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