Duval, Art M.

Free resolutions of simplicial posets. (English) Zbl 0882.06004

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A simplicial poset is a finite poset with minimal element in which every interval is isomorphic to a Boolean lattice. This notion generalizes the face poset of a simplicial complex. Indeed, any simplicial poset may be realized as the face poset of a regular cell complex in which every cell is isomorphic to a simplex. R. Stanley defined a ring A_P associated with a simplicial poset P which generalizes the face ring (or Stanley-Reisner ring) of a simplicial complex. In the paper under review, the author extends several important calculations from face rings of simplicial complexes to face rings of simplicial posets. The *Betti polynomial* of module M is the generating function for the ranks of the terms in a minimal free resolution of M. The first main result of the paper calculates the Betti polynomial of A_P as a module over $k[V]$, the polynomial ring on the set V of vertices of P. The result is expressed in terms of ordinary (topological) Betti numbers of subcomplexes of the realization of P. The proof follows the general outline of the proof in the simplicial complex case [*M. Hochster*, "Cohen-Macauley rings, combinatorics, and simplicial complexes", in: Ring Theory II, Proc. Second Oklahoma Conference 1975, 171-223 (1977; Zbl 0351.13009)], but uses a refinement of the usual $\mathbb{Z}^n$-grading of the Koszul complex ($n = |V|$). A similar, but more complicated analysis leads to the second main result, a calculation of the cohomology $H^*(A_P) := H^*(\mathcal{K}(\mathbf{x}^\infty, A_P))$ of a certain chain complex $\mathcal{K}(\mathbf{x}^\infty, A_P)$ determined by combinatorially defined local cohomology can be used to compute depth of modules. Thus the author is able to prove that (i) the depth of A_P is a topological invariant, that is, depends only on the realization of P as a topological space, and (ii) the depth of A_P is equal to $1+m$, where m is maximal such that the m-skeleton of the realization of P is a Cohen-Macauley complex. These also generalize known properties of simplicial complexes. The article ends with a detailed example.

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