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One-dimensional graph perturbations of selfadjoint relations. (English) Zbl 0894.47021

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Summary: Let A be a selfadjoint operator (or a selfadjoint relation) in a Hilbert space $\mathfrak{H}$, let Z be a one-dimensional subspace of $\mathfrak{H}^2$ such that $A\cap Z=\{0,0\}$ and define $S=A\cap Z^*$. Then S is a closed, symmetric operator (or relation) with defect numbers $(1,1)$ and, conversely, each such S and a selfadjoint extension A are related in this way. This allows us to interpret the selfadjoint extensions of S in $\mathfrak{H}$ as one-dimensional graph perturbations of A. If $Z=\text{span}\{\varphi,\psi\}$, then the function $Q(l)=l[\varphi,\varphi]+[(A-l)^{-1}(l\varphi-\psi),\bar{l}\varphi-\psi]$, generated by A and the pair $\{\varphi,\psi\}$, is a Q-function of $S=A\cap Z^*$ and A. It belongs to the class $\mathbf{N}$ of Nevanlinna functions and essentially determines S and A. Calculation of the corresponding resolvent operators in the perturbation formula leads to Kreĭn's description of (the resolvents of) the selfadjoint extensions of S. The class $\mathbf{N}$ of Nevanlinna functions has subclasses $\mathbf{N}_1\supset\mathbf{N}_0\supset\mathbf{N}_{-1}\supset\mathbf{N}_{-2}$, each defined in terms of function-theoretic properties. We characterize the Q-functions belonging to each of these classes in terms of the pair $\{\varphi,\psi\}$. If $Q(l)$ belongs to the subclass $\mathbf{N}_k$, $k=1,0,-1,-2$, then all but one of the selfadjoint extensions of S have a Q-function in the same class, while the exceptional extension has a Q-function in $\mathbf{N}\backslash\mathbf{N}_1$.

In particular, if S is semibounded, the exceptional selfadjoint extension is precisely the Friedrichs extension. We consider our perturbation formula in the case where the Q-function $Q(l)$ belongs to the subclass $\mathbf{N}_k$, $k=1,0,-1,-2$, or if it is an exceptional function associated with this subclass. The resulting perturbation formulas are made explicit for the case that A or its orthogonal operator part is the multiplication operator in a Hilbert space $L^2(d\rho)$.

MSC:

47B25	Linear symmetric and selfadjoint operators (unbounded)
47A55	Perturbation theory of linear operators
47A57	Linear operator methods in interpolation, moment and extension problems
47A20	Dilations, extensions, compressions of linear operators
47B15	Hermitian and normal operators (spectral measures, functional calculus, etc.)

Keywords:

selfadjoint operator; selfadjoint relation; symmetric operator; defect numbers; selfadjoint extension; graph perturbations; Nevanlinna functions; Kreĭn's description; Q-functions; orthogonal operator part; multiplication operator

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