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Bifurcation and stability of families of hyperbolic vector fields in dimension three. (English)

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Ann. Inst. Henri Poincaré, Anal. Non Linéaire 14, No. 1, 119-142 (1997).

Let M be a $\mathcal{C}^\infty$ compact boundaryless 3-dimensional manifold and $\chi^\infty(M)$ denote the space of $\mathcal{C}^\infty$ vector fields on M. The object of the study is the space $\chi_1^\infty(M)$ of all $\mathcal{C}^\infty$ arcs $\xi : I = [-1,1] \to \chi^\infty(M)$. For $\xi \in \chi_1^\infty(M)$ we let $\xi = \{X_\mu\}$ where $X_\mu = \xi(\mu)$ for each $\mu \in I$. Hence, ξ is a one-parameter family of vector fields on M.

We say that $\{X_\mu\}$ is stable at $\overline{\mu} \in I$, if there exists a neighborhood $\mathfrak{U}$ of $\{X_\mu\}$ in $\chi_1^\infty(M)$ such that for each $\{Y_\mu\} \in \mathfrak{U}$, there is a parameter value $\widetilde{\mu} \in I$ near $\overline{\mu}$ and a homeomorphism $H : M \times \overline{I} \to M \times \widetilde{I}$ where $\overline{I}$, respectively $\widetilde{I}$, is a neighborhood of $\overline{\mu}$, respectively of $\widetilde{\mu}$, in I and $H(x,\mu) = (h_\mu(x), \rho(\mu))$, with $\rho : (\overline{I}, \overline{\mu}) \to (\widetilde{I}, \widetilde{\mu})$ a reparametrization and $h_\mu : M \to M$ is a topological equivalence between X_μ and $Y_{\rho(\mu)}$, and the map $\mu \mapsto h_\mu$ is continuous.

In the paper the authors investigate the conditions under which the family $\{X_\mu\}$ is stable at $\overline{\mu} \in I$. Roughly speaking, $\{X_\mu\}$ is stable at $\overline{\mu}$, if $\overline{\mu}$ is its first bifurcation value in I and for $\mu = \overline{\mu}$ the vector field $X_{\overline{\mu}}$ has one and only one orbit $\overline{\gamma}$ along which it is not locally stable. The authors consider only the cases in which $\overline{\gamma}$ is an orbit of the following type: (1) an isolated saddle-node singularity; (2) an isolated Hopf singularity; (3) an isolated flip periodic orbit; (4) an isolated saddle-node periodic orbit; (5) a flip periodic orbit arising from two hyperbolic periodic orbits inside a basic set.

This interesting paper is well written and organized.

Reviewer: A.Klíč (Praha)

MSC:

37G99 Local and nonlocal bifurcation theory for dynamical systems
34D30 Structural stability and analogous concepts of solutions to ordinary differential equations
37D99 Dynamical systems with hyperbolic behavior
37C85 Dynamics induced by group actions other than $\mathbb{Z}$ and $\mathbb{R}$, and $\mathbb{C}$

Keywords:

structural stability; hyperbolic vector field; spectral decomposition; bifurcation set

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