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Thin elastic shells. Asymptotic properties. (Coques élastiques minces. Propriétés asymptotiques.) (French) Zbl 0881.73001

Recherches en Mathématiques Appliquées. Paris: Masson. xix, 376 p. (1997).

This fundamental work, very mathematical and very complete, studies in depth the case of linear elastic shells which presents some peculiarities, often at the origin of numerical irregularities, often localized (lockings, stiffening ridges, particular boundary conditions, etc.).

The introductory part of the book presents differential geometry, partial differential equations and elliptic systems appropriate to the shell geometry (Douglas and Nirenberg systems). In addition, the authors discuss here the surface rigidity, detailed properties of elliptic, hyperbolic, parabolic, cylindrical, developable surfaces, etc., and geometric deformations of surfaces, in particular different cases of bending which lead to the concept of inhibited or uninhibited shell (in flexion).

Then follow five chapters on theoretical mechanics. In chapter 6, the asymptotic theory of thin elastic shells is developed from Koiter's classical model and from some scaling assumptions on the membrane or on the flexion deformations. When the thickness tends to zero, the convergence study leads again to the difference between the inhibited and uninhibited cases. In particular, the authors consider singular perturbations in the inhibited case which are imposed due to special loading conditions.

The study of inhibited and uninhibited shells give the matter of fully detailed chapters 7 and 8. The first one studies the case where the middle surface is hyperbolic, elliptic or parabolic, all with precise boundary conditions. The second one uses variational principles with Lagrange multipliers. Chapter 9 returns to the asymptotic study, but now starting from the three-dimensional theory. Then the preceding distinctions are again exploited.

Chapter 10 is a new very detailed study of free vibrations. It is shown, among many facts, that, when the thickness tends to zero, the solution tends to the vibrations of membrane form, but where an essential spectrum exists, namely points of the spectrum that correspond to non-isolated eigenvalues with finite multiplicity. This spectrum is the matter of a special study. Moreover, the inhibited case is particular and leads to the case called by the authors "mean frequencies". Here, the limit method is no more possible. The problem of the eigenvalue convergence is thoroughly examined together with the "spectral pollution" phenomenon (cf. numerical computations).

The whole book provides a considerable number of results, and some valuable explanations of normal or abnormal behavior. However, in spite of unquestionable efforts, practical meaning of some hypotheses or conditions are sometimes absent.

The last chapter 11 investigates the membrane locking by the approximation by the finite element method for unhibited thin (inextentional) shells. The authors discuss an approximation method (robust case) such that the uniform convergence with the thickness tending to zero occurs which modifies the mesh in some cases. The difficulties of the polyhedral approximation make conspicuous the rigidification due to the ridges and due to the spectral pollution, although with a good approximation of vibration properties linked to the existence of essential spectra, except in some simple cases. Here an open problem exists.

Reviewer: R.Valid (Paris)

MSC:

74-02 Research exposition (monographs, survey articles) pertaining to mechanics of deformable solids
74K15 Membranes
35Q72 Other PDE from mechanics (MSC2000)
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

Koiter's model; inhibited shells; isolated eigenvalues with finite multiplicity; spectral pollution; differ-

ential geometry; elliptic systems; surface rigidity; uninhibited shell; convergence; singular perturbations; variational principles; Lagrange multipliers; free vibrations; spectrum; eigenvalue convergence; membrane locking; finite element method; polyhedral approximation