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The relationships between equilibria and positive solutions of certain nonlinear elliptic systems. (English) Zbl 0884.35032

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Author's abstract: Let $M(u,v)=0$, $N(u,v)=0$ define two distinct phase curves Γ_1, Γ_2 in the (u,v)-phase plane. This paper presents results on the relationships among the positive equilibria, the phase curves, and the existence of positive solutions to the PDE system

$$\Delta u + uM(u,v)=0, \quad \Delta v + vN(u,v)=0 \quad \text{in } \Omega \subset \mathbb{R}^n$$

under Dirichlet boundary conditions, where Ω is a bounded domain and M, N are monotone functions.

Reviewer: C.Y.Chan (Lafayette)

MSC:

35J55 Systems of elliptic equations, boundary value problems (MSC2000)
35J60 Nonlinear elliptic equations
92D25 Population dynamics (general)

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