Avallone, Anna; Weber, Hans
Lattice uniformities generated by filters. (English) Zbl 0907.06015
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In an earlier paper [Ann. Mat. Pura Appl. 160, 347-370 (1991; Zbl 0790.06006) and ibid. 165, 133-158 (1993; Zbl 0799.06014)], the second author studied uniform lattices as a common generalization of topological Boolean rings and topological Riesz spaces. Lattice uniformities play a similar role in the study of modular functions as FN-topologies.

The main aim of this paper is to give a contribution to the examination of the lattice structure of the space $\mathcal{LU}_c(L)$ of all exhaustive lattice uniformities on a lattice L. In particular, in the case where L is a modular sectionally complemented lattice, we get a satisfactory result; in this case $\mathcal{LU}_c(L)$ is a complete Boolean algebra (see Corollary 5.11). Such a result can be used as the main tool in obtaining decomposition theorems for modular functions as well as information about the structure of uniform completions of L.

Important for our investigations is the fact that a lattice uniformity on a sectionally complemented lattice L is uniquely determined by its 0-neighbourhood system; moreover the filters which are 0-neighbourhood filters for lattice uniformities are precisely the "distributive" filters, the space of which we indicate by $\mathcal{FND}(L)$.

We study this space $\mathcal{FND}(L)$ also for an arbitrary lattice L. Every filter of $\mathcal{FND}(L)$ induces on L lattice uniformity in a similar way that a distributive ideal in L induces a congruence relation. In fact, the concept of "distributive" filters introduced here and some statements about them were suggested by the concept and some properties of distributive ideals.

The main result essentialy says that, for a Hausdorff exhaustive lattice uniformity w on L generated by some $\mathcal{F}_0 \in \mathcal{FND}(L)$, the space of all lattice uniformities generated by filters of $\mathcal{FND}(L)$ coarser than $\mathcal{F}_0$ is isomorphic to the space of all distributive elements of the completion of (L, w). As a consequence, we get the result 5.11 mentioned above and the fact proved earlier [*H. Weber*, Order 12, 295-305 (1995; Zbl 0834.06013)] that the space of all exhaustive lattice uniformities on an orthomodular lattice is a complete Boolean algebra.

MSC:

06F30 Ordered topological structures (aspects of ordered structures)
54E15 Uniform structures and generalizations
54H12 Topological lattices, etc. (topological aspects)

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distributive filters; space of exhaustive lattice uniformities; decomposition; modular functions; uniform completions

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References:

[1] Basile, A.; Traynor, T., Monotonely Cauchy locally solid topologies, Order, 7, 407-416, (1991) · Zbl 0778.46007
[2] Beran, L., Orthomodular lattices, Mathematics and its applications, (1985), Reidel Dordrecht · Zbl 0558.06008
[3] Birkhoff, G., Lattice theory, (1967), Am. Math. Soc Providence · Zbl 0126.03801
[4] Drewnowski, L., Topological rings of sets, continuous set functions, integration I, II, and III, Bull. acad. polon. sci. ser. sci. math. astr. phys., 20, 269-276, (1972) · Zbl 0249.28004
[5] Grätzer, G., General lattice theory, Pure and applied mathematics series, (1978), Academic Press San Diego
[6] Maeda, F.; Maeda, S., Theory of symmetric lattices, (1970), Springer-Verlag Berlin/Heidelberg/New York · Zbl 0219.06002
[7] Waelbroecks, Topological vector spaces and algebras, Lecture notes in mathematics, 230, (1971), Springer-Verlag Berlin/New York
[8] Weber, H., Group- and vector-valued\textit{s}, Measure theory, Lecture notes in mathematics, 1089, (1984), Springer-Verlag

Berlin/New York, p. 181-198

[9] Weber, H., Uniform lattice: A generalization of topological Riesz spaces and topological Boolean rings; uniform lattice II: order continuity and exhaustivity, Ann. mat. pura appl., 160, 347-370, (1991) · Zbl 0790.06006

[10] Weber, H., Lattice uniformities and modular functions on orthomodular lattices, Order, 12, 295-305, (1995) · Zbl 0834.06013

[11] H. Weber, On modular functions, Funct. Approx. · Zbl 0887.06011

[12] H. Weber