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Convolutions equations containing singular probability distributions. (English. Russian original) Zbl 0882.45002

Izv. Math. 60, No. 2, 251-279 (1996); translation from Izv. Ross. Akad. Nauk, Ser. Mat. 60, No. 2, 21-48 (1996).

Let $V_C[a,b]$, $-\infty \le a < b \le \infty$, be the Banach space of continuous functions of bounded variation. The article is devoted to equations of the form

$$\varphi(x) = g(x) - \int_0^\infty \varphi(t) dT(x-t),$$

where T is in $V_C(R)$ and contains a singular component. In particular the following classes of operators are considered: $\Omega_C^+ = \{U^+\varphi(x) = -\int_0^x \varphi(t)dT(x-t) : T \in V_C[0,\infty]\}$, $\Omega_C^- = \{U^+\varphi(x) = \int_x^\infty \varphi(t)dT(t-x) : T \in V_C^+\}$, $\Omega_C = \{T_\varphi(x) = -\int_0^\infty \varphi(t)dT(x-t) : T \in V_C(R)\}$.

The author introduces and studies nonlinear factorization equations for T, i.e. equations of the form $I - T = (I - U^-)(I - U^+)$, where T is a given operator in Ω_C and $U\pm$ are operators in $\Omega_C^\pm$ to find. Factorization is constructed in the case when $T(-\infty) = 0$, $T(x) \uparrow$ in x, and $T(+\infty) = \mu \le 1$. With the aid of this factorization, existence theorems are proved for homogeneous ($g = 0$) and non-homogeneous equations in the singular case $\mu = 1$. Asymptotic and other properties of the solutions of formal Volterra equations corresponding to $T(x) = 0$ for $x \le 0$ are also investigated.

Reviewer: K.Georgiev (Rostov-na-Donu)

MSC:

45E10 Integral equations of the convolution type (Abel, Picard, Toeplitz and Wiener-Hopf type)

47B35 Toeplitz operators, Hankel operators, Wiener-Hopf operators

47A68 Factorization theory (including Wiener-Hopf and spectral factorizations) of linear operators

Keywords:

convolution equations; Wiener-Hopf operator; singular probability distribution; nonlinear factorization; existence

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