Stuart, C. A.
Bifurcation from the essential spectrum. (English) ⎡Zbl 0888.47045⎤
Matzeu, Michele (ed.) et al., Topological nonlinear analysis II: degree, singularity and variations. Papers presented in part at the 2nd topological analysis workshop on degree, singularity and variations: developments of the last 25 years, Frascati, Italy, June 1995. Boston, MA: Birkhäuser. Prog. Nonlinear Differ. Equ. Appl. 27, 397-443 (1997).

A survey of some of the results obtained during the past twenty years concerning bifurcation from a point of the essential spectrum of the linearization of a nonlinear equation is given. First, bifurcation problems

$$Au - N(u) = \lambda Lu \tag{1}$$

are considered where A, L are bounded linear selfadjoint operators in a Hilbert space, L is positive, $N = \nabla\varphi$, φ is a differentiable functional, $\lim_{|u|\to 0} \frac{\varphi(u)}{|u|^2} = 0$. General basic properties and relations of the discrete and essential spectrum to bifurcation points are recalled. Eigenvalue problems of the type

$$Su + R(u) = \lambda u$$

with an unbounded selfadjoint operator S in a real Hilbert space and a nonlinear perturbation R are simultaneously studied. It is shown how such problems can be cast in the form (1) under certain assumptions. Two types of results concerning bifurcation from the infimum Λ of the spectrum are explained. First, under certain assumptions, for any $\lambda < \Lambda$, a nontrivial u_λ satisfying (1) is found as a critical point of the functional $F(\lambda, u) = J(u) + \frac{1}{2}\lambda(Lu, u)$ where $J(u) = \frac{1}{2}(Au, u) - \varphi(u)$ and it is proved that $|u_\lambda| \to 0$ for $\lambda \to \Lambda$. Second, for any $r > 0$, a couple λ_r, u_r is found such that u_r is a stationary point of J subject to the constraint $(Lu, u) = r$, λ_r is a Lagrange multiplier. Further, general situations are described when there is a maximal interval (a, b) lying in the resolvent set of A, L and b is a bifurcation point. Moreover, there is a bifurcation to the left at b (bifurcation into spectral gaps). In some cases, also the order of the bifurcation is described. The last section is devoted to the problem

$$-\Delta u(x) + V(x)u(x) - r(x)|u(x)|^{p-2}u(x) = \lambda u(x) \text{ for } x \in \mathbf{R}^N$$

where $V \in \mathbf{L}^\infty(R^N)$, $r \in \mathbf{L}^\infty(R^N)$, $r \geq 0$ and $p < 2$ or $2 < p < \frac{2N}{N-2}$ in the case $N = 1, 2$ or $N \geq 3$, respectively. Known results about this problem are summarized. Particularly, general theory from previous sections is applied, i.e. bifurcations obtained by fixed λ approach, fixed norm approach and bifurcations into spectral gaps are described in various cases.

For the entire collection see [Zbl 0866.00047].

Reviewer: M.Kučera (Praha)

MSC:

47J10	Nonlinear spectral theory, nonlinear eigenvalue problems	
58C30	Fixed-point theorems on manifolds	
47A10	Spectrum, resolvent	
58E07	Variational problems in abstract bifurcation theory in infinite-dimensional spaces	
35B32	Bifurcations in context of PDEs	

Cited in **1** Review
Cited in **9** Documents

Keywords:

bifurcation; essential spectrum; infimum of the spectrum; spectral gap; semilinear elliptic equation; unbounded selfadjoint operator; nonlinear perturbation; Lagrange multiplier