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On shape from shading problem. (English) Zbl 0878.35026

Rassias, John M. (ed.), Functional analysis, approximation theory and numerical analysis. Dedicated to Stefan Banach on his 100th birthday, Alexander Markowiç Ostrowski on his 99th birthday, Stanislaw Marcin Ulam on his 83rd birthday. Singapore: World Scientific. 93-105 (1994).

The purpose of this note is to study the variational approach to the eikonal equation

$$\left(\frac{\partial u}{\partial x_1}\right)^2 + \left(\frac{\partial u}{\partial x_2}\right)^2 = \mathcal{E}(x_1, x_2) \tag{1}$$

in a bounded domain $\Omega \subset \mathbb{R}^2$. This equation arises in many branches of applied sciences. In particular, it appears in an area of computer vision in the so-called shape – from – shading problem in which one tries to solve the problem of how object shape can be recovered from image shading. We show that, in general, the variational approach does not lead to an exact solution of the shape – from – shading problem. In particular, we show that any function u satisfying the inequality $|Du(x)|^2 \leq \mathcal{E}(x)$ can be regarded as a "minimum" of the energy functional associated with (1) in the sense that there exists a suitable sequence $\{u_n\}$, with $u_n|_{\partial\Omega} = u|_{\partial\Omega}$, such that

$$\lim_{n\to\infty} \int_\Omega \left| |Du_n(x)|^2 - \mathcal{E}(x) \right| dx = 0 \quad \text{and} \quad u_n \rightharpoonup u \quad \text{weak} - * \text{ in } W^{1,\infty}(\Omega).$$

For the entire collection see [Zbl 0867.00015].

MSC:

35F20 Nonlinear first-order PDEs
35A15 Variational methods applied to PDEs

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eikonal equation; energy functional