Hamilton, Richard S.

Four-manifolds with positive isotropic curvature. (English) Zbl 0892.53018

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An incompressible space form N of a 4-manifold M is a 3-dimensional submanifold diffeomorphic to the quotient of a 3-sphere by a group G of linear isometries without fixed points, such that the fundamental group of $\pi_1(N)$ injects into $\pi_1(M)$. Such a space form is said to be essential unless $G = \{1\}$ or $G = \mathbb{Z}_2$ and the normal bundle is non-orientable.

In the paper under review, the author studies compact four-manifolds with no essential incompressible space forms. It is shown that M admits a metric of positive isotropic curvature if and only if the manifold is diffeomorphic to a sphere S^4, the projective space $\mathbb{R}P^4$, the product $S^3 \times S^1$, the nonoriented S^3 bundle over S^1, or a connected sum of the above. Positive isotropic curvature means that for all orthonormal vectors $\{e_1, e_2, e_3, e_4\}$, the curvature tensor satisfies

$$R_{1313} + R_{1414} + R_{2323} + R_{2424} \geq 2R_{1234}.$$

The result is proved by using the Ricci flow. The essential space forms defined above are obstructions for such a flow. If non-essential space forms exist, then a surgically modified Ricci flow is used to obtain the desired metric.

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MSC:

53C20	Global Riemannian geometry, including pinching	Cited in **8** Reviews
57N13	Topology of the Euclidean 4-space, 4-manifolds (MSC2010)	Cited in **52** Documents

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