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Fractals and spectra related to Fourier analysis and function spaces. (English) Zbl 0898.46030

Monographs in Mathematics. 91. Basel: Birkhäuser Verlag. viii, 271 p. (1997).

The book is devoted to study those aspects of fractal geometry in $\mathbb{R}^n$, which are connected to Fourier analysis, function spaces, and pseudodifferential operators. In the earlier book [*D. E. Edmunds* and *H. Triebel*, "Function spaces, entropy numbers, differential operators" (1996; Zbl 0865.46020)] the authors successfully applied estimates of entropy numbers of compact embeddings between function spaces to the spectral theory of degenerate pseudodifferential operators on bounded domains and on $\mathbb{R}^n$. A good part of the book under review is based on similar techniques, but this time in the context of fractals. The exposition departs from some basic material on fractals with special emphasis on the d-sets. One of the central aims of the book is to introduce and study function spaces on d-sets. Let Γ be a d-set. The $L^p(\Gamma)$ spaces are relatively easy to define since the measure on Γ is more or less uniquely determined, but their structure and relations to other function spaces are very complicated. This topic together with the introduction and study of the $B^s_{p,q}(\Gamma)$ spaces is treated in detail in Chapter 4, and it needs a lot of a deep preliminary material, which is contained in Chapters 2 and 3. These chapters include entropy numbers on weighted ℓ_p spaces with a dyadic block structure, and a new approach to the atomic decomposition of the spaces $B^s_{p,q}$ and $F^s_{p,q}$ on $\mathbb{R}^n$, consisting of further atomizing of the atoms, which results in subatomic (or quarkonial) decomposition. A thorough study of asymptotic behaviour of entropy numbers of embedding between these function spaces is carried out next. It is worth mentioning that there is virtually no literature on this topic and hence the most of the presented material is published here the first time. The final Chapter 5 deals with spectra of pseudodifferential operators with fractal coefficients. On suitable function spaces, these operators are compact, and estimates for distribution of their eigenvalues and counting function can be obtained. Particular attention is paid to the n-dimensional drums with a compact fractal layer.

Reviewer: L.Pick (Praha)

MSC:

46E35	Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems
28A80	Fractals
46-02	Research exposition (monographs, survey articles) pertaining to functional analysis
47G30	Pseudodifferential operators
47A10	Spectrum, resolvent

Cited in **12** Reviews
Cited in **117** Documents

Keywords:

fractals; function spaces; atomic decomposition; subatomic (quarkonial) decomposition; d-sets; approximation numbers; quadratic forms; pseudodifferential operators; fractal drums; Schrödinger operators; nonlinear elliptic equations related to fractals; fractal geometry; Fourier analysis; entropy numbers of compact embeddings; dyadic block structure

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