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Boundary effect for an elliptic Neumann problem with critical nonlinearity. (English)

Zbl 0891.35040
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Consider the Neumann problem

$$-\Delta u+\mu u=u^p \quad \text{in } \Omega, \quad u>0 \quad \text{in } \Omega, \quad \frac{\partial u}{\partial \nu}=0 \quad \text{on } \partial\Omega,$$

where $\mu>0$, Ω is smooth and bounded in $\mathbb{R}^n$, $n\geq 3$, and $p=(n+2)/(n-2)$ is the critical exponent. This paper describes some interactions between the boundary behavior of solutions to the Neumann problem and the mean curvature H of $\partial\Omega$ [see also *Adimurthi*, *F. Pacella*, and *S. L. Yadava*, J. Funct. Anal. 113, No. 2, 318-350 (1993; Zbl 0793.35033)], adapting the methods developed by the author for similar Dirichlet problems [J. Funct. Anal. 89, No. 1, 1-52 (1990; Zbl 0786.35059)].

Denote by H^b the level set of H to the level b and assume that the relative topology $(H^{a+\delta},H^{a-\delta})$ is non-trivial for a positive critical value a of H and for any $\delta>0$ sufficiently small. If $n\geq 5$, the author then proves the existence of a solution u_μ to the Neumann problem which concentrates, as μ tends to infinity, at some point $y\in\partial\Omega$ such that $H(y)=a$. Moreover, if $n\geq 6$ and $y^1,\dots,y^k$ are nondegenerate critical points of H with $H(y^j)>0$, there exists, for μ large enough, a solution u_μ which concentrates at $y^1,\dots,y^k$ (in the sense $|\nabla u_\mu|^2\rightharpoonup \frac{1}{2}S^{n/2}\sum_{i=1}^k\delta_{y^i}$, S being the best Sobolev constant for the embedding $H_0^1(\Omega)\hookrightarrow L^{2n/n-2}(\Omega)$). In this case, it is proved additionally that, for μ large enough, the Neumann problem admits at least 2^k-1 nonconstant solutions.

Reviewer: Ralf Beyerstedt (Triest)

MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
35J67 Boundary values of solutions to elliptic equations and elliptic systems
35B05 Oscillation, zeros of solutions, mean value theorems, etc. in context of PDEs

Keywords:

effect of boundary geometry; multiple solutions

Full Text: DOI

References:

[1] "A tribute in honour of G. Prodi" pp 9– (1991)

[2] in press.The role of the mean curvature in semilinear Neumann problem involving critical exponent,.Partial. Diff. Equ., · Zbl 0847.35047

[3] 182 (1989)

[4] in press.On a variational problem with lack of compactness: the topological effect of the critical points at infinity,.Calculus of Variations and Partial Diff. Equ.,

[5] 1991. "Asymptotic behaviour of solutions of elliptic equations with critical exponent and Neumann boundary conditions,". Vol.117 A, 225–250. Edinburgh Proc. Roy. Soc. · Zbl 0733.35038

[6] 1986. "On the Diffusion Coefficient of a Semilinear Neumann Problem,". Vol.1340, New York, Berlin: Springer Lecture Notes.

[7] 1982. "Models of Biological Pattern Formation,". New York, London: Academic Press.