Johnson, R. S.

A modern introduction to the mathematical theory of water waves. (English) Zbl 0892.76001

Cambridge Texts in Applied Mathematics. Cambridge: Cambridge University Press. xiv, 445 p. (1997).

The book is an introduction to the theory of water waves. It begins with the derivation of Euler equations governing the flow of inviscid fluid and with the formulation of free boundary conditions that define and describe the surface of the fluid. The equations and boundary conditions are rewritten in the nondimensional form to incorporate two small parameters: shallowness and amplitude. With these parameters at hand, it is possible to scale the flow variables and to yield various approximate versions of the governing equations and boundary conditions.

The technique adopted for handling approximate equations involves the construction of asymptotic expansions. After mathematical preliminaries, the presentation discusses some classical results on the linear water wave theory. Exact formulas are obtained for the speed of gravity and capillary waves over constant depth, and it is proved that the fluid particles follow the closed paths. The results also include the effect of variable depth and the ship wave patterns, together with the application of the ray theory to problems where the background flow varies slowly.

Passing to the nonlinear theory, the author first concentrates on classical results. This area includes the Stokes waves, nonlinear long waves via the method of characteristics (and Riemann invariants), the hydraulic jump and bore, waves on a sloping beach and the solitary waves. A large part of the volume is devoted to the modern aspects: problems that give rise to soliton-type equations. These are, first, the equations that belong to the Korteweg-de Vries family. Some relevant results from the soliton theory are quoted, and these are used to help in the interpretation of various equations and solutions that arise. The applications include the effects of shear and variable depth. Then the family of nonlinear Schrödinger equations is discussed in a similar fashion. The final chapter provides a brief introduction to the effects of viscosity.

All the mathematical developments are presented in the most straightforward manner, which is due to the use of asymptotic methods for the solution of differential equations. The approach does not require deep knowledge of the subject. Given a water wave problem, the process is initiated by assuming that a solution exists as a suitable asymptotic expansion with respect to the relevant small parameter. The form of this expansion is governed by the way in which the parameter appears in the equations and boundary conditions, and, a rather simple iterative construction suggests how this expansion proceeds. Exercises and further reading are provided at the end of each chapter, together with historical notes on some important topics.

Reviewer: Vladimir Shelukhin (Rio de Janeiro)

MSC:

76-02 Research exposition (monographs, survey articles) pertaining to fluid mechanics
35Q35 PDEs in connection with fluid mechanics
35Q51 Soliton equations

Keywords:

perfect conductor; Euler equations; free boundary conditions; shallowness; amplitude; approximate equations; asymptotic expansions; linear water wave theory; effect of variable depth; ship wave; ray theory; Stokes waves; method of characteristics; Riemann invariants; hydraulic jump; sloping beach; solitary waves; effects of viscosity; small parameters

Full Text: DOI