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Self-improving properties of John-Nirenberg and Poincaré inequalities on spaces of homogeneous type. (English) Zbl 0892.43005

J. Funct. Anal. 153, No. 1, 108-146 (1998).

The authors consider inequalities of the form

$$\fint_{\mu(B)} \int_B |f - f_B| d\mu \leq ca(B) \quad \text{and} \quad \fint_{\mu(B)} \int_B |f - f_B| d\mu \leq cb(B, f).$$

In either case μ is a measure and $\mu(B)$ denotes the μ-measure of B. The main goal of this paper is to show that under certain conditions of geometric type on the functionals a and b both inequalities encode an intrinsic L^r self-improving property.

Reviewer: Bolis Basit (Clayton)

MSC:

43A85 Harmonic analysis on homogeneous spaces

Cited in **5** Reviews
Cited in **44** Documents

Keywords:

self-improving properties of John-Nirenberg; Poincaré inequalities; spaces of homogeneous type

Full Text: DOI

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