Bassi, F.; **Rebay, S.**

High-order accurate discontinuous finite element solution of the 2D Euler equations. (English) Zbl 0902.76056

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Summary: This paper deals with a high-order accurate discontinuous finite element method for the numerical solution of the Euler equations. The method combines two key ideas, the basis of the finite volume and of the finite element method, the physics of wave propagation being accounted for by means Riemann problems and accuracy being obtained by means of high-order polynomial approximations within elements. We focus our attention on two-dimensional steady-state problems and present higher order accurate (up to fourth-order) discontinuous finite element solutions on unstructured grids of triangles. In particular, we show that, in the presence of curved boundaries, a meaningful high-order accurate solution can be obtained only if a corresponding high-order approximation of the geometry is employed. We present numerical solutions of classical test cases computed with linear, quadratic, and cubic elements which illustrate the versatility of the method and the importance of the boundary condition treatment.

MSC:

76M10 Finite element methods applied to problems in fluid mechanics
76N10 Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics

Keywords:

wave propagation; Riemann problems; high-order polynomial approximations; unstructured grids of triangles; curved boundaries

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