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A note on norm attaining functionals. (English) Zbl 0894.46011
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Summary: We are concerned with the density of functionals which do not attain their norms in Banach spaces. Some earlier results given for separable spaces are extended to the nonseparable case. We obtain that a Banach space X is reflexive if and only if it satisfies any of the following properties:

- (i) X admits a norm $\|\cdot\|$ with the Mazur intersection property and the set $NA_{\|\cdot\|}$ of all norm attaining functionals of X^* contains an open set,
- (ii) the set $NA_{\|\cdot\|}^1$ of all norm one elements of $NA_{\|\cdot\|}$ contains a (relative) weak* open set of the unit sphere,
- (iii) X^* has C^*PCP and $NA_{\|\cdot\|}^1$ contains a (relative) weak open set of the unit sphere,
- (iv) X is WCG , X^* has $CPCP$ and $NA_{\|\cdot\|}^1$ contains a (relative) weak open set of the unit sphere.

Finally, if X is separable, then X is reflexive if and only if $NA_{\|\cdot\|}^1$ contains a (relative) weak open set of the unit sphere.

MSC:

46B20 Geometry and structure of normed linear spaces

Cited in 13 Documents

Keywords:

reflexive spaces; Mazur intersection property; (weak*) convex point of continuity property; norm attaining functionals; C^*PCP ; WCG ; $CPCP$

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