Bey, Jürgen
Finite volume and multigrid methods for elliptic boundary value problems. (Finite-Volumen- und Mehrgitterverfahren für elliptische Randwertprobleme.) (German)
Zbl 0906.65120
Advances in Numerical Mathematics. Stuttgart: Teubner. 356 S. (1998).

One of the essential tools in solving linear systems originated by the discretization of partial differential equations is the use of adaptive multilevel methods. The recent book discusses – thoroughly – many of the problems in the scope of finite volume methods and of multilevel techniques.

Starting with an introduction into the theory and the numerics of elliptic boundary value problems, the author treats adaptive refinement algorithms, including a new technique for tetrahedral grid refinement, in this way completing his former, tentative results [Computing 55, No. 4, 355-378 (1995; Zbl 0839.65135)]. A careful examination of that technique is given. Further, the method is generalized to higher dimensions (> 3), based on an elder investigation of *H. Freudenthal* [Ann. Math. 43, 580-582 (1942; Zbl 0060.40701)], for which a detailed description (and complete proofs) are given together with a proof of its "optimality".

A further topic considered here is the finite-volume-discretization of n-dimensional elliptic boundary value problems (or box schemes, i.e. the application of finite differences on irregular nets). The treatment is a generalization of that due to *W. Hackbusch* [Computing 41, 277-296 (1989; Zbl 0649.65052)]. Finally the book handles a class of convection-diffusion problems (understood as singularly perturbed problems). Here the application of the methods proposed in the preceding chapters yields a (new) robust multilevel technique, whose core is a local Gauss-Seidel method with a special downwind numbering. Some examples (experiments) and sketches of programs make clear the ideas, especially in the construction of grids and their refinement.

Summarizing the items of the book: the plentyful content is in front of the recent research (and the way of expressing up to date, too).

Reviewer: E.Lanckau (Chemnitz)

MSC:

65N55	Multigrid methods; domain decomposition for boundary value problems involving PDEs	Cited in **5** Documents
65N06	Finite difference methods for boundary value problems involving PDEs	
65N30	Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs	
65N50	Mesh generation, refinement, and adaptive methods for boundary value problems involving PDEs	
65-02	Research exposition (monographs, survey articles) pertaining to numerical analysis	
35J40	Boundary value problems for higher-order elliptic equations	

Keywords:

research monographs (numerical analysis); domain decomposition (BVP of PDE); higher-order elliptic equations; multigrid methods; boundary value problems; adaptive multilevel methods; finite volume methods; adaptive refinement algorithms; tetrahedral grid refinement; finite differences; convection-diffusion problems; local-Gauss-Seidel method; downwind numbering