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Optimal comparison strategies in Ulam's searching game with two errors. (English)

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Summary: Suppose x is an n-bit integer. By a comparison question we mean a question of the form "does x satisfy either condition $a < x < b$ or $c < x < d$?". We describe strategies to find x using the smallest possible number $q(n)$ of comparison questions, and allowing up to two of the answers to be erroneous. As proved in this self-contained paper, with the exception of $n = 2$, $q(n)$ is the smallest number q satisfying Berlekamp's inequality $2^q \geqslant 2^n(\binom{q}{2} + q + 1)$. This result would disappear if we only allowed questions of the form "does x satisfy the condition $a < x < b$?". Since no strategy can find the unknown $x \in \{0, 1, \ldots, 2^n - 1\}$ with less than $q(n)$ questions, our result provides extremely simple optimal searching strategies for Ulam's game with two lies – the game of Twenty Questions where up to two of the answers may be erroneous.

MSC:

91A46 Combinatorial games

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