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Collisions of Calogero-Moser particles and an adelic Grassmannian (with an appendix by I. G. Macdonald). (English) Zbl 0906.35089

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A rational solution of the KP equation is known to correspond to a rational Calogero-Moser system, which is a classical particle system with inverse square potential. This paper reformulates such a correspondence in a geometrical way. Let $\overline{C}_n$ be the space of all pairs (X, Z) of $n \times n$ complex matrices such that $[X, Z] + I$ has rank one, and let C_n be the quotient space $\overline{C}_n / GL(n, \mathbb{C})$, where $GL(n, \mathbb{C})$ acts on $\overline{C}_n$ by simultaneous conjugation of X and Z. Let C'_n be the subspace of C_n represented by pairs (X, Z) with X diagonalizable. Then C'_n can be identified with the phase space for classical n-particle systems.

With this identification the Calogero-Moser flows on C'_n are quotients of certain $GL(n, \mathbb{C})$-invariant flows that are defined on the whole of $\overline{C}_n$, and C_n can be regarded as the completed phase space for the complex Calogero-Moser system. On the other hand, the rational solutions to the KP hierarchy are parametrized by a certain adelic Grassmannian $\mathrm{Gr}^{\mathrm{ad}}$, and there is a map from each C'_n into $\mathrm{Gr}^{\mathrm{ad}}$ commuting with the Calogero-Moser and KP flows of these spaces.

In this paper the author proves that these maps extend to a bijection $\beta : \bigcup_{n \geq 0} C_n \to \mathrm{Gr}^{\mathrm{ad}}$ and that the image of C_n under β is the union of all the open cells of dimension n in $\mathrm{Gr}^{\mathrm{ad}}$. The paper has also an appendix by I. G. Macdonald which gives an elementary proof of a formula for Schur functions.

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MSC:

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14M15 Grassmannians, Schubert varieties, flag manifolds

Keywords:

KP equations; Calogero-Moser systems; Schur functions

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